MATH 544, 1998, EXAM 4

PRINT Your Name:______________________________

There are 9 problems on 4 pages. Problem 1 is worth 20 points. Each of the other problems is worth 10 points. SHOW your work. *CIRCLE* your answer. **CHECK** your answer whenever possible. No Calculators.

1. Let

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix} \quad \text{and} \quad b = \begin{bmatrix} 5 \\ 2 \\ 1 \\ 4 \end{bmatrix}.$$

(It might be to your advantage to notice that the columns of A form an orthogonal set.)
(a) Find a matrix B so that BA is equal to the 3×3 identity matrix.
(b) Solve $Ax = b$.

2. Define "column space". Use complete sentences.

3. Define "linear transformation". Use complete sentences.

4. True or False. If the statement is true, then PROVE the statement. If the statement is false, then give a COUNTEREXAMPLE. If A and B are 2×2 nonsingular matrices, then $A + B$ is a nonsingular matrix.

5. True or False. If the statement is true, then PROVE the statement. If the statement is false, then give a COUNTEREXAMPLE. If U and V are subspaces of $\mathbb{R}^n$, then the intersection of U and V is also a subspace of $\mathbb{R}^n$.

6. Let W be the subspace of $\mathbb{R}^4$ which is spanned by

$$w_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \quad w_2 = \begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \end{bmatrix}, \quad w_3 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix}.$$

Find an orthogonal set which forms a basis for W.

7. Is

$$\left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \middle| \, x_1 x_2 = x_3 \right\}$$

a vector space? If so, explain why. If not, give an example to show that one of the rules of vector space fails to hold.

8. Is the function $F\colon \mathbb{R}^2 \to \mathbb{R}^2$, which is defined by

$$F\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \right) = \begin{bmatrix} x_1^2 \\ x_1 x_2 \end{bmatrix},$$

a linear transformation? If so, explain why. If not, give an example to show that one of the rules of linear transformation fails to hold.

9. Is the function $F\colon \mathbb{R}^2 \to \mathbb{R}^3$, which is defined by

$$F\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 \\ x_2 \\ 0 \end{bmatrix},$$

a linear transformation? If so, explain why. If not, give an example to show that one of the rules of linear transformation fails to hold.