

MATH 544, 1998, EXAM 3

PRINT Your Name: \_\_\_\_\_

There are 9 problems on 4 pages. Problem 3 is worth 20 points. Each of the other problems is worth 10 points. SHOW your work. **CIRCLE** your answer. **CHECK** your answer whenever possible. No Calculators.

1. Find the inverse of

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}.$$

2. Define “null space”. Use complete sentences.

3. Let

$$A = \begin{bmatrix} 1 & 2 & 1 & 0 \\ 2 & 5 & 3 & -1 \\ 2 & 2 & 0 & 2 \\ 0 & 1 & 1 & -1 \end{bmatrix}$$

- (a) Find a basis for the null space of  $A$ .
- (b) Find a basis for the column space of  $A$ .
- (c) Find a basis for the row space of  $A$ .

4. Define “basis”. Use complete sentences.

5. True or False. If the statement is true, then PROVE the statement. If the statement is false, then give a COUNTEREXAMPLE. If  $A$  and  $B$  are  $2 \times 2$  matrices, then

$$\text{the null space of } A \cap \text{the null space of } B \subseteq \text{the null space of } A + B.$$

6. True or False. If the statement is true, then PROVE the statement. If the statement is false, then give a COUNTEREXAMPLE. If  $A$  and  $B$  are  $2 \times 2$  nonsingular matrices, then  $A + B$  is a nonsingular matrix.

7. True or False. If the statement is true, then PROVE the statement. If the statement is false, then give a COUNTEREXAMPLE. If  $A$  and  $B$  are  $2 \times 2$  nonsingular matrices, then  $AB$  is a nonsingular matrix.

8. Let  $W$  be the subspace of  $\mathbb{R}^4$  which is spanned by

$$\begin{bmatrix} 1 \\ 3 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 2 \\ 2 \end{bmatrix}.$$

Find a basis for  $W$ .

9. True or False. If the statement is true, then PROVE the statement. If the statement is false, then give a COUNTEREXAMPLE. If  $U$  and  $V$  are subspaces of  $\mathbb{R}^n$ , then the union of  $U$  and  $V$  is also a subspace of  $\mathbb{R}^n$ .