544 Exam 2 1998

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Each Problem is worth 10 Points.

1) Define "linearly dependent".

2) Define "non-singular".

3) True or False. If true, Prove it. If false, give a counterexample.

If A and B are 2×2 matrices, then (the null space of A) $\cap$ (null space of B) $\subseteq$ null space of $A+B$.

4) True or False. If true, prove it. If false, give a counterexample.

If A and B are singular 2×2 matrices, then $A+B$ is a singular matrix.

5) Find all values a for which the system has no solution

$$\begin{aligned} 2x_1 + 4x_2 &= a \\ 3x_1 + 6x_2 &= 5 \end{aligned}$$

6) Solve the following system of equations:

$$\begin{aligned} x_1 + x_2 - x_5 &= 1 \\ x_2 + 2x_3 + x_4 + 3x_5 &= 1 \\ x_1 - x_3 + x_4 + x_5 &= 0 \end{aligned}$$

7) Is $\left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \middle| \ x_1 x_2 = x_3 \right\}$ a vector space? If so, explain why. If not, give an example to show that one of the rules for vector space fails to hold.

8) Let a and be be fixed vectors in $\mathbb{R}^3$.

Let $W = \left\{ x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in \mathbb{R}^3 \ \middle| \ a^T x = 0 \text{ and } b^T x = 0 \right\}$

Is W a vector space? If so, explain why. If not, give an