MATH 544, 1998, EXAM 1

PRINT Your Name:________________________________

There are 9 problems on 4 pages. Problem 1 is worth 12 points. Each of the other problems is worth 11 points. SHOW your work. $\boxed{\textit{CIRCLE}}$ your answer. **CHECK** your answer whenever possible.

1. Solve the following system of equations:

$$\begin{array}{rrrrrl} x_1 & +\, x_2 & & & -\, x_5 & = 1 \\ & x_2 & +2x_3 & +\, x_4 & +3x_5 & = 1 \\ x_1 & & -\, x_3 & +\, x_4 & +\, x_5 & = 0 \end{array}$$

2. Find all values of a for which the following system has no solution:

$$\begin{array}{rrl} x_1 & +2x_2 & = -3 \\ ax_1 & -2x_2 & = 5 \end{array}$$

3. Solve $Ax = b$ for

$$\begin{bmatrix} 1 & 2 & 3 & 0 & 4 & 5 & 0 \\ 0 & 0 & 0 & 1 & 6 & 7 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad \text{and} \quad b = \begin{bmatrix} 8 \\ 9 \\ 10 \end{bmatrix}$$

4. Solve $Ax = b$ for

$$\begin{bmatrix} 1 & 2 & 3 & 0 & 4 & 5 & 0 \\ 0 & 0 & 0 & 1 & 6 & 7 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{and} \quad b = \begin{bmatrix} 8 \\ 9 \\ 10 \end{bmatrix}$$

5. Compute

$$\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

6. Find scalars a_1 and a_2 so that $a_1 r + a_2 s = t$, where

$$r = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad s = \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \quad \text{and} \quad t = \begin{bmatrix} 1 \\ 4 \end{bmatrix}.$$

7. Find x so that $x^{\mathrm{T}} a = 6$ and $x^{\mathrm{T}} b = 2$, where

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad a = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad \text{and} \quad b = \begin{bmatrix} 3 \\ 4 \end{bmatrix}.$$

8. True or False. If the statement is true, then PROVE the statement. If the statement is false, then give a COUNTEREXAMPLE. If A and B are 2×2 symmetric matrices, then AB is a symmetric matrix.

9. True or False. If the statement is true, then PROVE the statement. If the statement is false, then give a COUNTEREXAMPLE. If A and B are 2×2 matrices with $A^2 = AB$, then $A = B$.