MATH 544, 1997, FINAL EXAM

PRINT Your Name:________________________________

There are 18 problems on 7 pages. Problem 1 is worth 14 points. Each of the other problems is worth 8 points. SHOW your work. *CIRCLE* your answer. **CHECK** your answer whenever possible. **NO CALCULATORS.**

1. Let A be an $n \times n$ matrix. List 8 statements that are equivalent to the statement " A is nonsingular".

2. Define "linear transformation".

3. Define "null space".

4. Define "span".

5. Let V be the vector space of polynomials $f(x)$ of degree at most three with $f(1) = 0$. Record a basis for V. No justification is needed.

Let

$$A = \begin{bmatrix} 1 & 2 & 2 & 6 & 2 & 8 \\ 1 & 2 & 3 & 9 & 2 & 8 \\ 1 & 2 & 3 & 9 & 3 & 12 \\ 2 & 4 & 5 & 15 & 5 & 20 \end{bmatrix} \quad \text{and} \quad b = \begin{bmatrix} 3 \\ 2 \\ 4 \\ 7 \end{bmatrix}.$$

6. Find a basis for the row space of A.

7. Find a basis for the column space of A.

8. Find a basis for the null space of A.

9. Solve $Ax = b$.

Let

$$A = \begin{bmatrix} \frac{5}{2} & \frac{3}{2} \\ \frac{3}{2} & \frac{5}{2} \end{bmatrix}.$$

10. Find an invertible matrix S and a diagonal matrix D with $S^{-1}AS = D$.

11. Find a matrix B with $B^2 = A$.

12. Let A be a symmetric matrix and let u and v be eigenvectors of A which belong to different eigenvalues. PROVE that $u^{\mathrm{T}}v = 0$.

13. True or False. If the statement is true, then PROVE the statement. If the statement is false, then give a COUNTEREXAMPLE. If A and B are 2×2 matrcies with A non-singular, then the column space of AB is equal to the column space of B.

14. True or False. If the statement is true, then PROVE the statement. If the statement is false, then give a COUNTEREXAMPLE. If A and B are 2×2 symmetric matrices, then AB is a symmetric matrix.

15. True or False. If the statement is true, then PROVE the statement. If the statement is false, then give a COUNTEREXAMPLE. If A and B are 2×2 nonsingular matrices, then AB is a nonsingular matrix.

16. True or False. If the statement is true, then PROVE the statement. If the statement is false, then give a COUNTEREXAMPLE. If A and B are 2×2 nonsingular matrices, then $A + B$ is a nonsingular matrix.

17. Find an orthogonal set which is a basis for the null space of $\begin{bmatrix} 1 & 2 & 1 & 2 \end{bmatrix}$.