MATH 544, 1997, EXAM 3

PRINT Your Name:_____

There are 4 pages. The problems are numbered from 1 to 8. The exam is worth 100 points. SHOW your work. $\boxed{CIRCLE}$ your answer. **CHECK** your answer whenever possible.

1. Let
$$A = \begin{bmatrix} 1 & 0 & 2 & 3 & 4 & 0 & 5 & 0 \\ 1 & 0 & 2 & 3 & 4 & 0 & 11 & 0 \\ 1 & 0 & 2 & 3 & 4 & 0 & 11 & 1 \end{bmatrix}.$$
   (a) (10 points) Find a basis for the null space of A.
   (b) (10 points) Find a basis for the column space of A.

2. Consider the vectors
$$u_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \quad u_2 = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}, \quad u_3 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix}, \quad \text{and} \quad u_4 = \begin{bmatrix} -1 \\ 1 \\ -1 \\ 1 \end{bmatrix}.$$
   (a) (2 points) Do the vectors u_1, u_2, u_3, u_4 form an orthogonal set? Why?
   (b) (9 points) Express $v = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$ as a linear combination of u_1, u_2, u_3, u_4.
   (c) (9 points) Find the inverse of $\begin{bmatrix} 1 & 1 & 0 & -1 \\ 1 & 0 & 1 & 1 \\ 1 & -1 & 0 & -1 \\ 1 & 0 & -1 & 1 \end{bmatrix}$.

3. (10 points) Define "basis".

4. (10 points) True or False. If the statement is true, then PROVE the statement. If the statement is false, then give a COUNTEREXAMPLE. If A and B are 2×2 matrcies with A non-singular, then the column space of AB is equal to the column space of B.

5. (10 points) True or False. If the statement is true, then PROVE the statement. If the statement is false, then give a COUNTEREXAMPLE. If A and B are 2×2 matrcies with A non-singular, then the null space of AB is equal to the null space of B.

6. (10 points) Define "null space".

7. (10 points) True or False. If the statement is true, then PROVE the statement. If the statement is false, then give a COUNTEREXAMPLE. If A and B are 2×2 matrcies with A non-singular, then the column space of BA is equal to the column space of B.

8. (10 points) True or False. If the statement is true, then PROVE the statement. If the statement is false, then give a COUNTEREXAMPLE. If A and B are 2×2 matrcies with A non-singular, then the null space of BA is equal to the null space of B.