

PRINT Your Name: _____

Quiz for March 18, 2010

The quiz is worth 5 points. **Remove EVERYTHING from your desk except this quiz and a pen or pencil. SHOW every step.** Express your work in a neat and coherent manner. BOX your answer.

Find a particular solution of $y'' - y' - 6y = 2 \sin 3x$.

ANSWER: We look for a solution of the form $y = A \sin 3x + B \cos 3x$. We compute

$$\begin{aligned}y' &= 3A \cos 3x - 3B \sin 3x \\y'' &= -9A \sin 3x - 9B \cos 3x.\end{aligned}$$

Plug our candidate into the DE to get

$$\left. \begin{aligned}-9A \sin 3x - 9B \cos 3x - (-3B \sin 3x + 3A \cos 3x) \\ -6(A \sin 3x + B \cos 3x)\end{aligned} \right\} = 2 \sin 3x.$$

We want

$$(-9A + 3B - 6A) \sin 3x + (-9B - 3A - 6B) \cos 3x = 2 \sin 3x.$$

We want

$$\begin{aligned}-15A + 3B &= 2 \\ -3A - 15B &= 0.\end{aligned}$$

We want

$$\begin{aligned}-5A + B &= 2/3 \\ -A - 5B &= 0.\end{aligned}$$

We want

$$\begin{bmatrix} -5 & 1 \\ -1 & -5 \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} = [2/3/0].$$

Multiply both sides of the equation on the left by $\begin{bmatrix} -5 & -1 \\ 1 & -5 \end{bmatrix}$. We want

$$\begin{bmatrix} -5 & -1 \\ 1 & -5 \end{bmatrix} \begin{bmatrix} -5 & 1 \\ -1 & -5 \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} -5 & -1 \\ 1 & -5 \end{bmatrix} \begin{bmatrix} 2/3 \\ 0 \end{bmatrix}.$$

We want

$$\begin{bmatrix} 26 & 0 \\ 0 & 26 \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} -10/3 \\ 2/3 \end{bmatrix}.$$

Take $A = \frac{-10}{3} \frac{1}{26} = \frac{-5}{39}$ and $B = \frac{2}{3} \frac{1}{26} = \frac{1}{39}$. Our answer is

$$\boxed{y = \frac{-5}{39} \sin 3x + \frac{1}{39} \cos 3x}.$$