

PRINT Your Name: \_\_\_\_\_

**Quiz for February 23, 2010**

The quiz is worth 5 points. **Remove EVERYTHING from your desk except this quiz and a pen or pencil. SHOW every step.** Express your work in a neat and coherent manner. BOX your answer.

Solve  $(x^2 + 1)\frac{dy}{dx} + 3xy = 6x$ . Express your answer in the form  $y(x)$ . **Check your answer.**

**ANSWER:** This is a first order linear DE:

$$\frac{dy}{dx} + \frac{3x}{x^2 + 1}y = \frac{6x}{x^2 + 1}.$$

The integrating factor is

$$e^{\int \frac{3x}{x^2+1} dx} = e^{3/2 \ln(x^2+1)} = (x^2 + 1)^{3/2}.$$

Multiply both sides by the integrating factor:

$$(x^2 + 1)^{3/2} \frac{dy}{dx} + 3x(x^2 + 1)^{1/2}y = 6x(x^2 + 1)^{1/2}.$$

$$\frac{d}{dx}((x^2 + 1)^{3/2}y) = 6x(x^2 + 1)^{1/2}.$$

Integrate both sides:

$$(x^2 + 1)^{3/2}y = 2(x^2 + 1)^{3/2} + C$$

$$\boxed{y = 2 + C(x^2 + 1)^{-3/2}}$$

**Check.** We see that  $dy/dx = -(3/2)C(x^2 + 1)^{-5/2}2x = \frac{-3Cx}{(x^2+1)^{5/2}}$ . So,

$$(x^2 + 1)\frac{dy}{dx} + 3xy = (x^2 + 1)\frac{-3Cx}{(x^2 + 1)^{5/2}} + 3x(2 + C(x^2 + 1)^{-3/2}) = 6x \checkmark$$