$$[\ln|x-4| - \ln|x-1|] = 3t + 3C$$

$$\left|\frac{x-4}{x-1}\right| = e^{3C}e^{3t}$$

$$\frac{x-4}{x-1} = \pm e^{3C}e^{3t}$$

Let K be the constant $\pm e^{3C}$; so,

$$\frac{x-4}{x-1} = Ke^{3t}.$$

Plug in $t=0$ to see that $K = \frac{x_0-4}{x_0-1}$. Multiply both sides by $x-1$ to see

$$x - 4 = Ke^{3t}(x-1).$$

Add $-Ke^{3t}x$ to both sides to obtain

$$x - Ke^{3t}x = 4 - Ke^{3t}.$$

So,

$$x = \frac{4 - Ke^{3t}}{1 - Ke^{3t}}$$

$$x = \frac{4 - \left(\frac{x_0-4}{x_0-1}\right)e^{3t}}{1 - \left(\frac{x_0-4}{x_0-1}\right)e^{3t}}$$

$$\boxed{x(t) = \frac{4(x_0-1) - (x_0-4)e^{3t}}{(x_0-1) - (x_0-4)e^{3t}}} \quad \text{answer to (a).}$$

(c) If $4 < x_0$, then the denominator of $x(t)$ becomes zero at the positive time $t = (1/3)\ln\left(\frac{x_0-1}{x_0-4}\right)$. In other words, if $4 < x_0$, then $x(t)$ goes to infinity in finite time.

(d) If $x_0 < 4$, then the denominator of $x(t)$ is positive for all non-negative t . In this case,

$$\lim_{x\to\infty} x(t) = \lim_{x\to\infty} \frac{4(x_0-1) - (x_0-4)e^{3t}}{(x_0-1) - (x_0-4)e^{3t}} = \lim_{x\to\infty} \frac{4(x_0-1)e^{-3t} - (x_0-4)}{(x_0-1)e^{-3t} - (x_0-4)}$$

$$= \frac{x_0-4}{x_0-4} = 1.$$