Write everything on the blank paper provided.

You should KEEP this piece of paper.

If possible: turn the problems in order (use as much paper as necessary), use only one side of each piece of paper, and leave 1 square inch in the upper left hand corner for the staple. If you forget some of these requests, don't worry about it – I will still grade your exam.

The exam is worth 50 points. There are **5** problems. Each problem is worth 10 points.

SHOW your work. $\boxed{CIRCLE}$ your answer. Write **coherently**.

No Calculators or Cell phones.

I will post the solutions later today.

1. **Consider the Initial Value Problem** $\frac{dx}{dt} = (x-1)(x-4)$, $x(0) = x_0$.
   (a) **Solve the Initial Value Problem.**
   (b) **Draw some of the solutions.**
   (c) **Which choices for x_0 cause x to go to infinity at some finite time?**
   (d) **Which choices for x_0 cause x to go toward a finite constant as t goes to infinity.**

We see that if

$$\frac{1}{(x-1)(x-4)} = \frac{A}{x-1} + \frac{B}{x-4},$$

then $1 = A(x-4) + B(x-1)$. Plug in $x = 1$ to see that $A = -1/3$. Plug in $x = 4$ to see $B = 1/3$. We check that

$$\frac{1}{3}\left(\frac{1}{x-4} - \frac{1}{x-1}\right) = \frac{1}{(x-1)(x-4)}.$$

We solve the differential equation:

$$\int \frac{dx}{(x-1)(x-4)} = \int dt$$

$$\frac{1}{3}\int\left(\frac{1}{x-4} - \frac{1}{x-1}\right)dx = \int dt$$

$$\frac{1}{3}\left[\ln|x-4| - \ln|x-1|\right] = t + C$$