We are supposed to find t with $P(t) = 900$. We solve the following equation for t:

$$900 = \frac{90 \cdot 100}{100 + (90 - 100)e^{\frac{9t}{100}}}$$

$$1 = \frac{10}{100 + (90 - 100)e^{\frac{9t}{100}}}$$

$$100 + (90 - 100)e^{\frac{9t}{100}} = 10$$

$$-10e^{\frac{9t}{100}} = -90$$

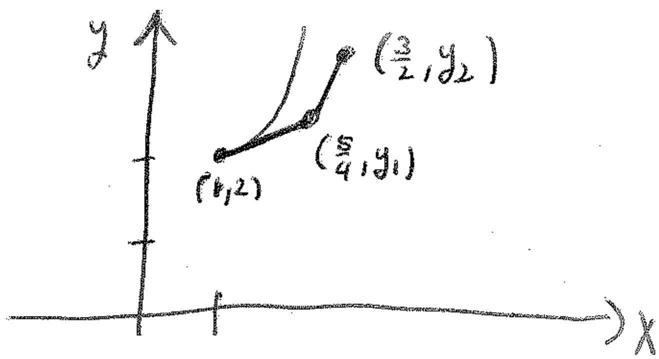
$$e^{\frac{9t}{100}} = 9$$

$$\frac{9t}{100} = \ln 9$$

The population hits 900 when the time is $\boxed{\frac{100}{9}\ln 9}$ months.

6. (6 points) **Consider the initial value problem $\frac{dy}{dx} = x + y^3$, $y(1) = 2$. Use Euler's method to approximate $y(3/2)$. Use two steps, each of size $1/4$.**

Consider the picture:



The curve represents the correct solution of the initial value problem. We approximate the real solution with the two line segments. The line segment from $(1, 2)$ to $(\frac{5}{4}, y_1)$ has slope equal to $(x + y^3)|_{(1,2)}$. The line segment from $(\frac{5}{4}, y_1)$ to $(\frac{3}{2}, y_2)$ has slope equal to $(x + y^3)|_{(\frac{5}{4}, y_1)}$. Of course, y_2 is our approximation of $y(\frac{3}{2})$.

We see that $(x + y^3)|_{(1,2)} = 9$; so, the segment from $(1, 2)$ to $(\frac{5}{4}, y_1)$ has slope equal to 9. This segment lives on the line

$$y - 2 = 9(x - 1),$$

which is $y = 9x - 7$. Thus, $y_1 = 45/4 - 7 = 17/4$.

The line segment from $(\frac{5}{4}, y_1)$ to $(3/2, y_2)$ has slope equal to

$$(x+y^3)|_{(\frac{5}{4},y_1)} = (x+y^3)|_{(\frac{5}{4},\frac{17}{4})} = \frac{5}{4} + \left(\frac{17}{4}\right)^3 = \frac{17^3+16(5)}{64}.$$

This segment lives on the line

$$y - \frac{17}{4} = \frac{17^3+16(5)}{64}\left(x - \frac{5}{4}\right)$$

which is

$$y = \frac{17^3+16(5)}{64}\left(x - \frac{5}{4}\right) + \frac{17}{4}$$

Our approximation of $y(3/2)$ is

$$\boxed{y_2 = \frac{17^3+16(5)}{64}\left(\frac{1}{4}\right) + \frac{17}{4}.}$$