Fall 1994 Exam 3 Math 242

PRINT Your Name:____________________________

There are 6 problems on 3 pages. Problems 3 and 4 are worth 9 points each. Each of the other problems is worth 8 points. The exam is worth a total of 50 points. SHOW your work. $\boxed{CIRCLE}$ your answer. **CHECK** your answer, whenever possible.

1. Find all solutions of $y'' = (y')^2$.

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$$\frac{dy'}{(y')^2} = dx$$

$$-\frac{1}{y'} = x + A$$

$$y' = \frac{-1}{x+A}$$

$$y = -\ln|x+A| + B$$

2. Find all solutions of $x^2y'' + 3xy' - 3y = 0$.

8 $y = x^r \quad y' = rx^{r-1} \quad y'' = r(r-1)x^{r-2}$

$$r(r-1) + 3r - 3 = 0$$

$$r^2 + 2r - 3 = 0$$

$$(r+3)(r-1) = 0$$

$$y = c_1 x^{-3} + c_2 x$$

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3. Find ONE solution of $y'' + 2y' + 2y = \cos x$.

$y = A\cos x + B\sin x$

$y' = -A\sin x + B\cos x$

$y'' = -A\cos x - B\sin x$

$-A\cos x - B\sin x$

$+2B\cos x - 2A\sin x$

$2A\cos x + 2B\sin x = \cos x$

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$A + 2B = 1 \qquad B - 2A = 0$

$A = \frac{1}{5}$

$B = \frac{2}{5}$

$y = \frac{1}{5}\cos x + \frac{2}{5}\sin x$

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4. Find ONE solution of $y'' + y = \sec x$.

$y = u_1 \sin x + u_2 \cos x$ where

$$\begin{bmatrix} \sin x & \cos x \\ \cos x & -\sin x \end{bmatrix} \begin{bmatrix} u_1' \\ u_2' \end{bmatrix} = \begin{bmatrix} 0 \\ \sec x \end{bmatrix}$$

$$\begin{bmatrix} u_1' \\ u_2' \end{bmatrix} = - \begin{bmatrix} -\sin x & -\cos x \\ -\cos x & \sin x \end{bmatrix} \begin{bmatrix} 0 \\ \sec x \end{bmatrix} = \begin{bmatrix} 1 \\ -\tan x \end{bmatrix}$$

$u_1 = x$

$u_2 = +\ln|\cos x|$

$y = x\sin x + \cos x \ln|\cos x|$

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5. Find $\mathcal{L}^{-1}\left(\dfrac{s^2-2s}{s^4+5s^2+4}\right)$.

$$= \frac{1}{3}\left[\mathcal{L}^{-1}\left(\frac{2s+4}{s^2+4}\right) + \mathcal{L}^{-1}\left(\frac{-2s-1}{s^2+1}\right)\right]$$

$$= \frac{2}{3}\cos 2t + \frac{2}{3}\sin 2t - \frac{1}{3}\sin t - \frac{2}{3}\cos t$$

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6. Find $\mathcal{L}(f(t))$ for

$$f(t) = \begin{cases} t & \text{if } 0 \le t \le 1, \text{ and} \\ 0 & \text{if } 1 < t. \end{cases}$$

$$\mathcal{L}(f) = \int_0^1 t e^{-st} dt = -\frac{1}{s} t e^{-st}\Big]_0^1 + \frac{1}{s}\int_0^1 e^{-st} dt$$

$u = t \quad v = -\frac{1}{s}e^{-st}$

$du = dt \quad dv = e^{-st} dt$

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$$= -\frac{1}{s}\left(\frac{1}{e^s}\right) + \frac{1}{s}\left(-\frac{1}{s}\right)e^{-st}\Big]_0^1$$

$$= \frac{-1}{se^s} - \frac{1}{s^2e^s} + \frac{1}{s^2}$$

$$= \frac{1}{s^2}\left[1 - \frac{1}{e^s} - \frac{s}{e^s}\right]$$

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