# Math 242 Fall 1994 Exam 2

PRINT Your Name:______________________________

There are 6 problems on 3 pages. Problems 3 and 4 are worth 9 points each. Each of the other problems is worth 8 points. The exam is worth a total of 50 points. SHOW your work. $\boxed{CIRCLE}$ your answer. **CHECK** your answer, whenever possible.

1. State the Existence and Uniqueness Theorem for linear second order differential equations.

Consider the Initial Value Problem

$$(*)\begin{cases} y'' + P_1(x)y' + P_0(x)y = Q(x) \\ y(a) = b_0, \quad y'(a) = b_1. \end{cases}$$

If $P_1(x)$, $P_2(x)$, and $Q(x)$ are continuous on some interval $I$ which contains $x=a$, then $(*)$ has a unique solution which is defined on all of $I$.

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2. Find all solutions of $y'' - 2y' + y = 0$.

$r^2 - 2r + 1 = 0$

$(r-1)^2 = 0$

8

$y = c_1 e^x + c_2 x e^x$

8

3. Find all solutions of $y'' + y = 15e^{2x}$.

The solution of $y''+y=0$ is $y = c_1 \cos x + c_2 \sin x$

Try $y = Ae^{2x}$

$y' = 2Ae^{2x}$

$y'' = 4Ae^{2x}$

$4Ae^{2x} + Ae^{2x} = 15e^{2x}$

$5A = 15$

$A = 3$

9

$y = c_1 \cos x + c_2 \sin x + 3e^{2x}$

4. Find all solutions of $y'' - y = 4e^{x}$.

The solution of $y''-y=0$ is $y = c_1 e^{x} + c_2 e^{-x}$

Try $y = Axe^{x}$

$y' = Axe^{x} + Ae^{x}$

$y'' = Axe^{x} + 2Ae^{x}$

$Axe^{x} + 2Ae^{x} - Axe^{x} = 4e^{x}$

$2A = 4$

$A = 2$

9

$y = c_1 e^{x} + c_2 e^{-x} + 2xe^{x}$

9

5. Find all solutions of $x^3 + 3y - xy' = 0$.

$y' - \frac{3}{x} y = x^2$

$\mu = e^{\int -\frac{3}{x} dx} = \frac{1}{x^3}$

$\frac{1}{x^3} y' - \frac{3}{x^4} y = \frac{1}{x}$

8

$\frac{1}{x^3} y = \ln x + C$

$y = x^3(\ln x + C)$

6. Find all solutions of $xy + y^2 - x^2 y' = 0$.

$\frac{y}{x} + \left(\frac{y}{x}\right)^2 = y'$

$v = \frac{y}{x}$

$xv = y$

8

$xv' + v = y'$

$\cancel{v} + v^2 = xv' + \cancel{x}$

$\frac{dx}{x} = \frac{dv}{v^2}$

$\ln x + c = -\frac{1}{v}$

$-\frac{y}{x} = \frac{1}{\ln x + C}$

$y = \frac{-x}{\ln x + C}$

$1 - n$

$v = y^{-1}$

$v' = -\underline{y^{-2}} y'$

$xy^{-1} + 1 - x^2 y' y^{-2} = 0$

$xv + 1 + x^2 v' = 0$

$v' + \frac{1}{x} v = -\frac{1}{x^2}$

$\mu = e^{\int \frac{1}{x} dx} = x$

$xv' + v = -\frac{1}{x}$

$xv = -\ln x + C$

$\frac{1}{y} = v = \frac{C - \ln x}{x}$

$\frac{x}{C - \ln x} = y$

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