1993 Exam 2 Math 242

① $r^2-3r+2=0 \quad (r-1)(r-2)=0 \quad \boxed{y = c_1 e^{x} + c_2 e^{2x}}$

② $r^2-2r+1=0 \quad (r-1)^2=0 \quad \boxed{y = c_1 e^{x} + c_2 x e^{x}}$

③ $r^2-4r+13=0 \quad r = \dfrac{+4 \pm \sqrt{16-4\cdot 13}}{2} = \dfrac{4 \pm 2\sqrt{4-13}}{2}$

$r = 2 \pm 3i \qquad \boxed{y = e^{2x}\left(c_1 \cos 3x + c_2 \sin 3x\right)}$

④ $y' = y + y^3 \qquad v = y^{-2} \qquad v' = -2y^{-3}y'$

$-2y'y^{-3} = -2y^{-2} - 2$

$v' + 2v = -2$ let $\mu = e^{\int 2\,dx} = e^{2x}$

$v'e^{2x} + 2e^{2x}v = -2e^{2x}$

$ve^{2x} = -e^{2x} + c$

$v = -1 + ce^{-2x}$

$y^{-2} = -1 + ce^{-2x}$

$$\boxed{y = \frac{1}{\sqrt{ce^{-2x}-1}}}$$

⑤ $xy' = y + 2\sqrt{xy}$

$y' = \dfrac{y}{x} + 2\sqrt{\dfrac{y}{x}} \qquad$ Let $v = \dfrac{y}{x} \qquad xv = y$

$xv' + v = y'$

$xv' + \cancel{v} = \cancel{v} + 2\sqrt{v}$

$$\frac{dv}{\sqrt{v}} = \frac{2\,dx}{x}$$

$$2v^{\frac{1}{2}} = 2\ln|x| + c$$

$$v^{\frac{1}{2}} = \ln|x| + k \qquad \left(\text{where } k = \frac{c}{2}\right)$$

$$\left(\frac{y}{x}\right)^{\frac{1}{2}} = \ln|x| + k$$

$$\boxed{y = x\left(\ln|x| + k\right)^2}$$

(6) Consider the Initial Value Problem

$$y^{(n)} + P_1(x)\,y^{(n-1)} + \dots + P_{n-1}(x)\,y' + P_n(x) = Q(x)$$

$$y(a) = b_0, \ \dots, \ y^{(n-1)}(a) = b_{n-1}.$$

If $P_1, \dots, P_n, Q$ are all continuous on some interval $I$ about $x = a$, then the IVP has unique solution which is defined on all of $I$.

(7) $2x'' + 12x' + 50x = 0 \quad x(0) = 0 \quad x'(0) = -8$

$2r^2 + 12r + 50 = 2(r^2 + 6r + 25) = 0 \qquad r = \dfrac{-6 \pm \sqrt{36 - 100}}{2} = \dfrac{-6 \pm 8i}{2}$

$r = -3 \pm 4i \qquad x = e^{-3t}(c_1 \cos 4t + c_2 \sin 4t)$

$0 = x(0) = c_1 \qquad x' = e^{-3t} c_2 (\cos 4t)4 - 3e^{-3t} c_2 \sin 4t$

$-8 = x'(0) = c_2 \cdot 4 \qquad c_2 = -2$

$$\boxed{x(t) = -2e^{-3t} \sin 4t}$$