

Math 242, Spring 1994, Exam 1

Use your own paper. SHOW your work. CIRCLE your answer.

1. (15 points) State the Existence and Uniqueness Theorem for first order differential equations.
2. (25 points)
 - (a) What does the Existence and Uniqueness Theorem tell you about the Initial Value Problem

$$(1 + x^2)y' = (2 + y)^2 \quad y(0) = 0?$$

- (b) Solve the Initial Value Problem of part (a).
- (c) What does the Existence and Uniqueness Theorem tell you about the Initial Value Problem

$$(1 + x^2)y' = (2 + y)^2 \quad y(0) = -2?$$

- (d) Solve the Initial Value Problem of part (c).
3. (15 points) Solve the Initial Value Problem $y' = x \sin x$, $y(0) = 0$.
 4. (15 points) Solve the Initial Value Problem $xy' = 3y + x^4 \cos x$, $y(2\pi) = 0$.
 5. (15 points) A 400 gallon tank initially contains 100 gallons of brine containing 50 pounds of salt. Brine containing 1 pound of salt per gallon enters the tank at the rate of 5 gal./sec., and the mixed brine in the tank flows out at the rate the rate of 3 gal./sec.. How much salt will the tank contain at the moment it becomes full?
 6. (15 points) The rate of decay of a radioactive substance is proportional to the amount of the substance present. Two grams of a radioactive substance are left to decay. It is observed that after a year 1.4 grams are left. What will the amount be after a total time of 10 years?