Quiz Feb. 9, 2012

Solve $y' = y + y^3$

This is a Bernoulli Equation $y' - y = y^3$.

Let $v = y^{-2}$. so $\frac{dv}{dx} = -2y^{-3}\frac{dy}{dx}$

Solve

$$y^{-3}y' - y^{-2} = 1$$

$$-\frac{1}{2}\frac{dv}{dx} - v = 1$$

$$\frac{dv}{dx} + 2v = -2$$

$$\mu = e^{\int 2\,dx} = e^{2x}$$

$$e^{2x}\frac{dv}{dx} + 2e^{2x}v = -2e^{2x}$$

$$\frac{d}{dx}(e^{2x}v) = -2e^{2x}$$

$$\int \frac{d}{dx}(e^{2x}v)\,dv = -2\int e^{2x}dx$$

$$e^{2x}v = -e^{2x} + C$$

$$v = -1 + Ce^{-2x}$$

$$y^{-2} = -1 + Ce^{-2x}$$

$$\boxed{y = \frac{\pm 1}{\sqrt{Ce^{-2x} - 1}}}$$

we check

$$y = \frac{\pm 1}{\sqrt{Ce^{-2x} - 1}}$$

we have

$$y' = \frac{-2Ce^{-2x}}{-2(Ce^{-2x}-1)^{3/2}}$$

$$y' = \frac{Ce^{-2x}}{(Ce^{-2x}-1)^{3/2}}$$

$$y + y^3 = \frac{1}{\sqrt{Ce^{-2x}-1}} + \frac{1}{(Ce^{-2x}-1)^{\frac{3}{2}}}$$

$$= \frac{Ce^{-2x} - 1 + 1}{(Ce^{-2x}-1)^{\frac{3}{2}}}$$

$$= \frac{Ce^{-2x}}{(Ce^{-2x}-1)^{\frac{3}{2}}}$$

Thus $y' = y + y^3$, as desired.