Please PRINT your name ______________________________________

The quiz is worth 5 points. Please make your work coherent, complete, and correct. Please $\boxed{\textit{CIRCLE}}$ your answer. Please **CHECK** your answer whenever possible.

The solution will be posted later today.

No Calculators, computers, smart phones, notes, etc.

Quiz 8, April 17, 2018

Use the method of Laplace transforms to solve the Initial Value Problem

$$x'' + x = \cos 3t, \quad x(0) = 1,\ x'(0) = 0.$$

Answer:

Let $X = \mathcal{L}(x)$. We have $\mathcal{L}(x') = s\mathcal{L}(x) - x(0) = sX - 1$, $\mathcal{L}(x'') = s\mathcal{L}(x') - x'(0) = s^2X - s$, and $\mathcal{L}(\cos 3t) = \frac{s}{s^2+9}$. Transform the given IVP to

$$s^2X - s + X = \frac{s}{s^2+9}.$$

We solve for X:

$$(s^2+1)X = \frac{s}{s^2+9} + s$$

$$X = \frac{s}{(s^2+1)(s^2+9)} + \frac{s}{s^2+1}$$

Apply the technique of partial fractions to get

$$X = \frac{1}{8}\left[\frac{1}{s^2+1} - \frac{1}{s^2+9}\right] + \frac{s}{s^2+1}$$

$$X = \frac{1}{8}\left[9\frac{1}{s^2+1} - \frac{1}{s^2+9}\right]$$

Apply the inverse Laplace Transform:

$$x = \mathcal{L}^{-1}(X) = \mathcal{L}^{-1}\left(\frac{1}{8}\left[9\frac{1}{s^2+1} - \frac{1}{s^2+9}\right]\right) = \frac{1}{8}\left[9\cos t - \cos 3t\right].$$

Our answer is

$$\boxed{x(t) = \frac{1}{8}\left[9\cos t - \cos 3t\right].}$$