

PRINT Your Name: _____

Quiz 8, Fall, 2012 – October 30

The quiz is worth 5 points. **Remove EVERYTHING from your desk except this quiz and a pen or pencil.** SHOW your work. Express your work in a neat and coherent manner. BOX your answer.

1. Find the general solution of $6y^{(4)} + 5y^{(3)} + 25y'' + 20y' + 4y = 0$. You might find it helpful to know that $y = \cos 2x$ is a solution of the Differential Equation.

Answer. The hint tells us that $2i$ and $-2i$ are roots of the characteristic polynomial

$$6r^4 + 5r^3 + 25r^2 + 20r + 4.$$

It follows that $(r - 2i)(r + 2i) = r^2 + 4$ is a factor of the characteristic polynomial. The other factor is $6r^2 + 5r + 1$ and this may be found by performing long division:

$$6r^4 + 5r^3 + 25r^2 + 20r + 4 = (r^2 + 4)(6r^2 + 5r + 1) = (r^2 + 4)(2r + 1)(3r + 1).$$

The roots of the characteristic polynomial are $\pm 2i$, $-1/2$, and $-1/3$. The general solution of the Differential equation is

$$y = c_1 \cos 2x + c_2 \sin 2x + c_3 e^{-x/2} + c_4 e^{-x/3}.$$

Check We take derivatives of our proposed answer:

$$\begin{aligned} y' &= -2c_1 \sin 2x + 2c_2 \cos 2x - (1/2)c_3 e^{-x/2} - (1/3)c_4 e^{-x/3} \\ y'' &= -4c_1 \cos 2x - 4c_2 \sin 2x + (1/4)c_3 e^{-x/2} + (1/9)c_4 e^{-x/3} \\ y''' &= 8c_1 \sin 2x - 8c_2 \cos 2x - (1/8)c_3 e^{-x/2} - (1/27)c_4 e^{-x/3} \\ y'''' &= 16c_1 \cos 2x + 16c_2 \sin 2x + (1/16)c_3 e^{-x/2} + (1/81)c_4 e^{-x/3} \end{aligned}$$

Plug our proposed solution back into the DE

$$\begin{aligned} &6(16c_1 \cos 2x + 16c_2 \sin 2x + (1/16)c_3 e^{-x/2} + (1/81)c_4 e^{-x/3}) \\ &+ 5(8c_1 \sin 2x - 8c_2 \cos 2x - (1/8)c_3 e^{-x/2} - (1/27)c_4 e^{-x/3}) \\ &+ 25(-4c_1 \cos 2x - 4c_2 \sin 2x + (1/4)c_3 e^{-x/2} + (1/9)c_4 e^{-x/3}) \\ &+ 20(-2c_1 \sin 2x + 2c_2 \cos 2x - (1/2)c_3 e^{-x/2} - (1/3)c_4 e^{-x/3}) \\ &+ 4(c_1 \cos 2x + c_2 \sin 2x + c_3 e^{-x/2} + c_4 e^{-x/3}) \end{aligned}$$

$$= \begin{cases} c_1[(96 - 100 + 4) \cos 2x + (40 - 40) \sin 2x] \\ + c_2[(-40 + 40) \cos 2x + (96 - 100 + 4) \sin 2x] \\ + c_3\left(\frac{6-10+100-160+64}{16}\right)e^{-x/2} \\ + c_4\left(\frac{6-15+225-540+324}{81}\right)e^{-x/3} \end{cases}$$

$$= 0. \checkmark$$