PRINT Your Name:__________________________________

Quiz for June 13, 2012

The quiz is worth 5 points. **Remove EVERYTHING from your desk except this quiz and a pen or pencil.** SHOW your work. Express your work in a neat and coherent manner. $\boxed{\text{BOX}}$ your answer.

Solve $9y''' + 12y'' + 4y' = 0$.
Check your answer.

Answer. Try $y = e^{rx}$. Plug y into the DE; get $9r^3e^{rx} + 12r^2e^{rx} + 4re^{rx} = 0$. Factor this equation as $e^{rx}(9r^3 + 12r^2 + 4r) = 0$. Of course, e^{rx} is never 0; so, $9r^3 + 12r^2 + 4r = 0$ or $r(9r^2 + 12r + 4) = 0$ or $r(3r + 2)^2 = 0$. The roots of the characteristic polynomial are 0 with multiplicity 1 and $-\frac{2}{3}$ with multiplicity 2. It follows that e^{0x}, $e^{(-\frac{2}{3})x}$ and $y = xe^{(-\frac{2}{3})x}$ are three linearly independent solutions of the DE. Of course, e^{0x} is the same as 1. The general solution of the DE is

$$\boxed{y = c_1 + c_2e^{(-\frac{2}{3})x} + c_3xe^{(-\frac{2}{3})x}.}$$

Check. Plug the proposed answer back into the DE. We see that

$$\bullet\ y' = -\tfrac{2}{3}c_2e^{(-\frac{2}{3})x} + c_3(-\tfrac{2}{3}xe^{(-\frac{2}{3})x} + e^{(-\frac{2}{3})x}),$$

$$y'' = \tfrac{4}{9}c_2e^{(-\frac{2}{3})x} + c_3(\tfrac{4}{9}xe^{(-\frac{2}{3})x} - \tfrac{2}{3}e^{(-\frac{2}{3})x} - \tfrac{2}{3}e^{(-\frac{2}{3})x}),$$

$$\bullet\ y'' = \tfrac{4}{9}c_2e^{(-\frac{2}{3})x} + c_3(\tfrac{4}{9}xe^{(-\frac{2}{3})x} - 2(\tfrac{2}{3})e^{(-\frac{2}{3})x}),$$

$$y''' = -\tfrac{8}{27}c_2e^{(-\frac{2}{3})x} + c_3(-\tfrac{8}{27}xe^{(-\frac{2}{3})x} + \tfrac{4}{9}e^{(-\frac{2}{3})x} + 2(\tfrac{4}{9})e^{(-\frac{2}{3})x}),$$

and

$$\bullet\ y''' = -\tfrac{8}{27}c_2e^{(-\frac{2}{3})x} + c_3(-\tfrac{8}{27}xe^{(-\frac{2}{3})x} + 3(\tfrac{4}{9})e^{(-\frac{2}{3})x}).$$

Thus,

$$9y''' + 12y'' + 4y' = \begin{bmatrix} 9[-\frac{8}{27}c_2e^{(-\frac{2}{3})x} + c_3(-\frac{8}{27}xe^{(-\frac{2}{3})x} + 3(\frac{4}{9})e^{(-\frac{2}{3})x})] \\ +12[\frac{4}{9}c_2e^{(-\frac{2}{3})x} + c_3(\frac{4}{9}xe^{(-\frac{2}{3})x} - 2(\frac{2}{3})e^{(-\frac{2}{3})x})] \\ +4[-\frac{2}{3}c_2e^{(-\frac{2}{3})x} + c_3(-\frac{2}{3}xe^{(-\frac{2}{3})x} + e^{(-\frac{2}{3})x})] \end{bmatrix}$$

$$= c_2\left(\tfrac{-72+144-72}{27}\right)e^{(-\frac{2}{3})x} + c_3\left(\tfrac{-72+144-72}{27}\right)xe^{(-\frac{2}{3})x} + c_3(12 - 16 + 4)e^{(-\frac{2}{3})x} = 0. \checkmark$$