

Please PRINT your name _____

The quiz is worth 5 points. Please make your work coherent, complete, and correct. Please **CIRCLE** your answer. Please **CHECK** your answer whenever possible.

The solution will be posted later today.

No Calculators, computers, smart phones, notes, etc.

Quiz 6, March 27, 2018

Find the solution of the Initial Value Problem $y'' + 9y = \sin 2x$, $y(0) = 1$, $y'(0) = 0$.

Answer. Of course you know that the general solution of $y'' + 9y = 0$ is $y = c_1 \cos 3x + c_2 \sin 3x$. Also, it is easy to see that $y_{\text{particular}} = \frac{1}{5} \sin 2x$ is a particular solution of the given DE. It follows that the general solution of the DE $y'' + 9y = \sin 2x$ is $y = c_1 \cos 3x + c_2 \sin 3x + \frac{1}{5} \sin 2x$. We must find c_1 and c_2 so that the Initial Conditions $y(0) = 1$ and $y'(0) = 0$ are also satisfied. We compute $y' = -3c_1 \sin 3x + 3c_2 \cos 3x + \frac{2}{5} \cos 2x$. Plug $x = 0$ into y and y' to obtain:

$$1 = y(0) = c_1 \quad \text{and} \quad 0 = y'(0) = 3c_2 + \frac{2}{5}.$$

We conclude that $c_1 = 1$ and $c_2 = -\frac{2}{15}$. Thus the answer is

$$y = \cos 3x - \frac{2}{15} \sin 3x + \frac{1}{5} \sin 2x.$$

Check. We take derivatives of $y = \cos 3x - \frac{2}{15} \sin 3x + \frac{1}{5} \sin 2x$ to obtain $y' = -3 \sin 3x - \frac{2}{5} \cos 3x + \frac{2}{5} \cos 2x$ and $y'' = -9 \cos 3x + \frac{6}{5} \sin 3x - \frac{4}{5} \sin 2x$. It is clear that $y'' + 9y = 3 \sin 2x$. We plug 0 in for x to see that $y(0) = 1$ and $y'(0) = -\frac{2}{5} + \frac{2}{5} = 0$. ✓