Quiz 6, March 24, 2017, 1:15 class

Solve the spring problem $x'' + 8x' + 16x = 0$, $x(0) = 5$, $x'(0) = -10$. Put the answer in the form $x(t) = Ce^{-pt}\cos(\omega t - \alpha)$, if this makes sense.

Answer: This is a second order linear homogeneous DE with constant coefficients. We look for solutions of the form $x(t) = e^{rt}$. In other words, we consider the the characteristic polynomial

$$r^2 + 8r + 16 = 0$$
$$(r+4)^2 = 0$$

Thus, $x(t) = e^{-4t}$ and $x(t) = te^{-4t}$ are solutions of the DE. The general solution of the DE is

$$x(t) = e^{-4t}(c_1 + c_2 t).$$

We compute

$$x'(t) = e^{-4t}(c_2) - 4e^{4t}(c_1 + c_2 t).$$

Plug in $t = 0$ to evaluate the constants

$$5 = x(0) = c_1 \quad \text{and} \quad -10 = x'(0) = c_2 - 4c_1.$$

So, $c_1 = 5$ and $c_2 = 10$. The solution of the IVP is

$$\boxed{x(t) = 5e^{-4t}(1+2t)}.$$

Check: Plug

$$\begin{cases} x(t) = 5e^{-4t}(1+2t) \\ x'(t) = 5e^{-4t}(2) - 20e^{-4t}(1+2t) = e^{-4t}(-10-40t) \\ x''(t) = e^{-4t}(-40) - 4e^{-4t}(-10-40t) = 160te^{-4t} \end{cases}$$

into the DE to get

$$160te^{-4t} + 8(e^{-4t}(-10-40t)) + 16(5e^{-4t}(1+2t))$$
$$= e^{-4t}(160t - 80 - 320t + 80 + 160t) = 0. \checkmark$$

We also calculate $x(0) = 5 \checkmark$ and $x'(0) = -10 \checkmark$.