Please PRINT your name _____

The quiz is worth 5 points. Please make your work coherent, complete, and correct. Please $\boxed{CIRCLE}$ your answer. Please **CHECK** your answer whenever possible.

The solution will be posted later today.

No Calculators, computers, smart phones, notes, etc.

Quiz 5, March 20, 2018

Solve $9y''' + 12y'' + 4y' = 0$. **Check your answer.**

<u>Answer:</u> Try $y = e^{rx}$. Plug y into the DE; get $9r^3 e^{rx} + 12r^2 e^{rx} + 4re^{rx} = 0$. Factor this equation as $e^{rx}(9r^3 + 12r^2 + 4r) = 0$. Of course, e^{rx} is never 0; so, $9r^3 + 12r^2 + 4r = 0$ or $r(9r^2 + 12r + 4) = 0$ or $r(3r+2)^2 = 0$. The roots of the characteristic polynomial are 0 with multiplicity 1 and $-\frac{2}{3}$ with multiplicity 2. It follows that e^{0x}, $e^{(-\frac{2}{3})x}$ and $y = xe^{(-\frac{2}{3})x}$ are three linearlly independent solutions of the DE. Of course, e^{0x} is the same as 1. The general solution of the DE is

$$\boxed{y = c_1 + c_2 e^{(-\frac{2}{3})x} + c_3 x e^{(-\frac{2}{3})x}.}$$

Check. Plug the proposed answer back into the DE. We see that

$$\bullet \ y' = -\tfrac{2}{3}c_2 e^{(-\frac{2}{3})x} + c_3\left(-\tfrac{2}{3}xe^{(-\frac{2}{3})x} + e^{(-\frac{2}{3})x}\right),$$

$$y'' = \tfrac{4}{9}c_2 e^{(-\frac{2}{3})x} + c_3\left(\tfrac{4}{9}xe^{(-\frac{2}{3})x} - \tfrac{2}{3}e^{(-\frac{2}{3})x} - \tfrac{2}{3}e^{(-\frac{2}{3})x}\right),$$

$$\bullet \ y'' = \tfrac{4}{9}c_2 e^{(-\frac{2}{3})x} + c_3\left(\tfrac{4}{9}xe^{(-\frac{2}{3})x} - 2(\tfrac{2}{3})e^{(-\frac{2}{3})x}\right),$$

$$y''' = -\tfrac{8}{27}c_2 e^{(-\frac{2}{3})x} + c_3\left(-\tfrac{8}{27}xe^{(-\frac{2}{3})x} + \tfrac{4}{9}e^{(-\frac{2}{3})x} + 2(\tfrac{4}{9})e^{(-\frac{2}{3})x}\right),$$

and

$$\bullet \ y''' = -\tfrac{8}{27}c_2 e^{(-\frac{2}{3})x} + c_3\left(-\tfrac{8}{27}xe^{(-\frac{2}{3})x} + 3(\tfrac{4}{9})e^{(-\frac{2}{3})x}\right).$$

Thus,

$$9y''' + 12y'' + 4y' = \begin{bmatrix} 9[-\tfrac{8}{27}c_2 e^{(-\frac{2}{3})x} + c_3(-\tfrac{8}{27}xe^{(-\frac{2}{3})x} + 3(\tfrac{4}{9})e^{(-\frac{2}{3})x})] \\ +12[\tfrac{4}{9}c_2 e^{(-\frac{2}{3})x} + c_3(\tfrac{4}{9}xe^{(-\frac{2}{3})x} - 2(\tfrac{2}{3})e^{(-\frac{2}{3})x})] \\ +4[-\tfrac{2}{3}c_2 e^{(-\frac{2}{3})x} + c_3(-\tfrac{2}{3}xe^{(-\frac{2}{3})x} + e^{(-\frac{2}{3})x})] \end{bmatrix}$$

$$= c_2\left(\tfrac{-72+144-72}{27}\right)e^{(-\frac{2}{3})x} + c_3\left(\tfrac{-72+144-72}{27}\right)xe^{(-\frac{2}{3})x} + c_3(12 - 16 + 4)e^{(-\frac{2}{3})x} = 0. \ \checkmark$$