Please PRINT your name ______________________________

No calculators, cell phones, computers, notes, etc.

Circle your answer. Make your work correct, complete and coherent.

Please take a picture of your quiz (for your records) just before you turn the quiz in. I will e-mail your grade and my comments to you. I will keep your quiz.

The quiz is worth 5 points. The solutions will be posted on my website later today.

Quiz 5, November 1, 2023

Find a function y which satisfies the Differential Equation $y''' - 6y'' - 11y' - 6y = 0$ and the Initial conditions $y(0) = 0$, $y'(0) = 0$, and $y''(0) = 2$. Please notice that $y_1 = e^x$, $y_2 = e^{2x}$, and $y_3 = e^{3x}$ all are solutions of the Differential Equation[1] $y''' - 6y'' - 11y' - 6y = 0$.

ANSWER: The functions y_1, y_2, and y_3 are linearly independent. So the general solution of the third order linear DE with constant coefficients $y''' - 6y'' - 11y' - 6y = 0$ is $y = Ae^x + Be^{2x} + Ce^{3x}$. We find A, B, and C so that the initial conditions are satisfied. We compute $y' = Ae^x + 2Be^{2x} + 3Ce^{3x}$ and $y'' = Ae^x + 4Be^{2x} + 9Ce^{3x}$. We solve

$$\begin{cases} Ae^0 + Be^0 + Ce^0 = 0 \\ Ae^0 + 2Be^0 + 3Ce^0 = 0 \\ Ae^0 + 4Be^0 + 9Ce^0 = 3 \end{cases}$$

simultaneously. We solve

$$\begin{cases} A + B + C = 0 \\ A + 2B + 3C = 0 \\ A + 4B + 9C = 2 \end{cases}$$

simultaneously. Add minus eq1 to eq2 and minus eq1 to eq3. We solve

$$\begin{cases} A + B + C = 0 \\ \quad B + 2C = 0 \\ \quad 3B + 8C = 2 \end{cases}$$

simultaneously. Add -3 eq2 to eq3. We solve

$$\begin{cases} A + B + C = 0 \\ \quad B + 2C = 0 \\ \qquad 2C = 2 \end{cases}$$

We see that $C = 1$, $B = -2$, and $A = 1$. The solution is

$$\boxed{y = e^x - 2e^{2x} + e^{3x}.}$$

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[1] Actually there is a typo in the statement of this problem. It should be $y''' - 6y'' + 11y' - 6y = 0$. This typo does not affect anything in finding A, B, and C. However, none of the functions y_1, y_2, or y_3 are solutions of $y''' - 6y'' - 11y' - 6y = 0$; they all are solutions of $y''' - 6y'' + 11y' - 6y = 0$.