Please PRINT your name ______________________________

No calculators, cell phones, computers, notes, etc.

Circle your answer. Make your work correct, complete and coherent.

Please take a picture of your quiz (for your records) just before you turn the quiz in. I will e-mail your grade and my comments to you.

The quiz is worth 5 points. The solutions will be posted on my website later today.

Quiz 4, February 21, 2024

Let $P(t)$ represent the number of alligators in a certain park at time t. Suppose further that $P(t)$ satisfies the Differential Equation

$$\frac{dP}{dt} = \frac{B_0}{P_0^2}P^2 - \frac{D_0}{P_0}P,$$

where P_0 is the alligator population at time zero, B_0 is the birth rate at time zero, and D_0 is the death rate at time zero. The solution of the Differential Equation is

$$P(t) = \frac{M}{1 - \frac{P_0 - M}{P_0}e^{\frac{D_0 t}{P_0}}},$$

where $M = \frac{D_0 P_0}{B_0}$. **Use this value of $P(t)$. I do not expect you to solve the Differential equation or even to verify that the given solution is correct.**

Suppose that $P_0 = 100$ alligators, $B_0 = 10$ alligators per month, and $D_0 = 9$ alligators per month. When will the alligator population reach ten times M?

ANSWER: Calculate $M = \frac{D_0 P_0}{B_0} = \frac{9(100)}{10} = 90$. Observe that $P(t)$ is equal to ten times M when

$$900 = \frac{90}{1 - \frac{100-90}{100}e^{\frac{9t}{100}}};$$

In other words,

$$900 = \frac{90}{1 - \frac{1}{10}e^{\frac{9t}{100}}};$$

Multiply both sides by $1 - \frac{1}{10}e^{\frac{9t}{100}}$ and divide both sides by 900; obtain

$$1 - \frac{1}{10}e^{\frac{9t}{100}} = \frac{1}{10}.$$

Add $\frac{1}{10}e^{\frac{9t}{100}}$ to both sides and subtract $\frac{1}{10}$ from both sides:

$$\frac{9}{10} = \frac{1}{10}e^{\frac{9t}{100}}.$$

Multiply both sides by 10 and take the logarithm of both sides:

$$\ln 9 = \frac{9t}{100}.$$

Multiply both sides by $\frac{100}{9}$:

$$\frac{100}{9}\ln 9 = t.$$

The population will reach ten times M after $\frac{100}{9}\ln 9$ months.