

Please PRINT your name _____

No calculators, cell phones, computers, notes, etc.

Circle your answer. Make your work correct, complete and coherent.

Please take a picture of your quiz (for your records) just before you turn the quiz in. I will e-mail your grade and my comments to you. I will keep your quiz.

The quiz is worth 5 points. The solutions will be posted on my website later today.

Quiz 3, October 4, 2023

Solve the Differential Equation

$$\frac{dy}{dx} = (4x + y)^2.$$

Write your answer in the form $y = y(x)$. If you have time, please check that your answer is correct.

ANSWER: We make a linear substitution. Let $v = 4x + y$. It follows that $\frac{dv}{dx} = 4 + \frac{dy}{dx}$. We solve

$$\frac{dv}{dx} - 4 = v^2$$

by separating the variables:

$$\begin{aligned}\frac{dv}{dx} &= v^2 + 4 \\ \frac{dv}{v^2 + 4} &= dx\end{aligned}$$

Integrate both sides:

$$\begin{aligned}\int \frac{dv}{v^2 + 4} &= \int dx \\ \frac{1}{2} \arctan\left(\frac{v}{2}\right) &= x + C \\ \arctan\left(\frac{v}{2}\right) &= 2x + 2C\end{aligned}$$

(Let $K = 2C$.)

$$\begin{aligned}\frac{v}{2} &= \tan(2x + K) \\ v &= 2 \tan(2x + K) \\ 4x + y &= 2 \tan(2x + K) \\ \boxed{y = -4x + 2 \tan(2x + K)}.\end{aligned}$$

Check: We compute

$$\frac{dy}{dx} = -4 + 2 \sec^2(2x + K) \cdot 2 = 4(\sec^2(2x + K) - 1) = 4 \tan^2(2x + K)$$

On the other hand, $(4x + y)^2 = (2 \tan(2x + K))^2 = 4 \tan^2(2x + K)$. ✓