PRINT Your Name: _____

The quiz is worth 5 points. **Remove EVERYTHING from your desk except this quiz and a pen or pencil.** SHOW your work. Express your work in a neat and coherent manner. $\boxed{\text{BOX}}$ your answer. **Check your answer**

1. **Solve** $yy' + x = \sqrt{x^2 + y^2}$. **Express your answer in the form** $y(x)$. **Check your answer.**

This is a homogeneous problem. Divide both sides by x to write the problem as

$$\frac{y}{x}y' + 1 = \sqrt{1 + \left(\frac{y}{x}\right)^2}.$$

Let $v = \frac{y}{x}$. In other words, $xv = y$. Take the derivative with respect to x to see that $xv' + v = y'$. We must solve

$$v(xv' + v) + 1 = \sqrt{1 + v^2}.$$

We must solve

$$xv\frac{dv}{dx} = \sqrt{1 + v^2} - v^2 - 1.$$

We must solve

$$v\frac{dv}{\sqrt{1 + v^2} - v^2 - 1} = \frac{dx}{x}.$$

Integrate both sides. Let $w = 1 + v^2$. It follows that $dw = 2vdv$. We must solve

$$\frac{1}{2}\int \frac{dw}{\sqrt{w} - w} = \ln|x| + C.$$

We have

$$\ln|x| + C = \frac{1}{2}\int \frac{dw}{\sqrt{w}(1 - \sqrt{w})}.$$

Let $u = \sqrt{w}$. We have $du = \frac{1}{2}w^{-1/2}dw$.
   We have

$$\ln|x| + C = \int \frac{du}{1 - u} = -\ln|1 - u| = -\ln|1 - \sqrt{w}| = -\ln|1 - \sqrt{1 + v^2}|$$

$$= -\ln\left|1 - \sqrt{1 + \left(\frac{y}{x}\right)^2}\right| = -\ln\left|\frac{x - \sqrt{x^2 + y^2}}{x}\right| = -\ln\left|x - \sqrt{x^2 + y^2}\right| + \ln|x|.$$

Subtract $\ln|x|$ from both sides:

$$C = -\ln\left|x - \sqrt{x^2 + y^2}\right|$$

or

$$\ln\left|x - \sqrt{x^2 + y^2}\right| = -C.$$

Exponentiate. Let K be the new constant e^{-C}. We have

$$x - \sqrt{x^2 + y^2} = K;$$

so $x - K = \sqrt{x^2 + y^2}$ and $(x - K)^2 = x^2 + y^2$ and $\boxed{\pm\sqrt{(x - K)^2 - x^2} = y.}$

Check: We check $y = +\sqrt{(x - K)^2 - x^2}$, with $K \le x$. We see that

$$y' = \frac{2(x - K) - 2x}{2\sqrt{(x - K)^2 - x^2}} = \frac{-K}{\sqrt{(x - K)^2 - x^2}}.$$

So, $yy' + x = -K + x$. On the other hand,

$$\sqrt{x^2 + y^2} = \sqrt{x^2 + (x - K)^2 - x^2} = \sqrt{(x - K)^2} = x - K.$$

Thus, $yy' + x = \sqrt{y^2 + x^2}$ as required. ✓