PRINT Your Name:_____

Quiz 2, Spring, 2013

The quiz is worth 5 points. **Remove EVERYTHING from your desk except this quiz and a pen or pencil.** SHOW your work. Express your work in a neat and coherent manner. BOX your answer. **Check your answer.** The solution will be posted later today.

Solve the initial value problem $xy' = 3y + x^4 \cos x\,,\ y(2\pi) = 0$

ANSWER: This is a first order linear problem: $y' - \frac{3}{x}y = x^3 \cos x$. It has the form $y' + P(x)y = Q(x)$, where $P(x) = -\frac{3}{x}$ and $Q(x) = x^3 \cos x$. We multiply both sides by

$$e^{\int P(x)dx} = e^{\int \frac{-3}{x}dx} = e^{-3\ln x} = x^{-3}$$

to obtain

$$x^{-3}y' - 3x^{-4}y = \cos x.$$

Thus,

$$\frac{d}{dx}(x^{-3}y) = \cos x.$$

Integrate both sides with respect to x to obtain

$$x^{-3}y = \sin x + C.$$

Plug in 2π for x to see that

$$(2\pi)^{-3}0 = 0 + C;$$

so $C = 0$. The solution of this IVP is

$$\boxed{y = x^3 \sin x.}$$

Check. We see that $y(2\pi) = (2\pi)^3 \sin(2\pi) = 0$ as desired. Now we plug the proposed answer into the DE. The Left Hand Side, xy', becomes

$$x(x^3 \cos x + 3x^2 \sin x) = x^4 \cos x + 3x^3 \sin x).$$

The Right Hand Side, $3y + x^4 \cos x$, becomes:

$$3x^3 \sin x + x^4 \cos x.$$

These agree, so our proposed answer satisfies the DE.