Please PRINT your name _____

The quiz is worth 5 points. Please make your work coherent, complete, and correct. Please $\boxed{CIRCLE}$ your answer. Please **CHECK** your answer whenever possible.

The solution will be posted later today.

No Calculators, computers, smart phones, notes, etc.

Quiz 1, January 23, 2018

Find a function $y = f(x)$ which solves the differential equation with the prescribed initial condition:

$$\frac{dy}{dx} = xe^{-x} \quad \text{and} \quad y(0) = 1.$$

Answer: We compute

$$y = \int \frac{dy}{dx} dx = \int xe^{-x} dx.$$

Use integration by parts. Let $u = x$ and $dv = e^{-x} dx$. Compute $du = dx$ and $v = -e^{-x}$. The integration by parts formula is

$$\int u \, dv = uv - \int v \, du.$$

Thus,

$$y = -xe^{-x} + \int e^{-x} dx = -xe^{-x} - e^{-x} + C.$$

We check this much before going further:

$$\frac{dy}{dx} = xe^{-x} - e^{-x} + e^{-x} = xe^{-x},$$

as expected. Now we evaluate the constant:

$$1 = y(0) = -1 + C.$$

So $C = 2$ and $\boxed{y = -xe^{-x} - e^{-x} + 2.}$