

PRINT Your Name: \_\_\_\_\_

**Quiz 1, Fall, 2016**

The quiz is worth 5 points. **Remove EVERYTHING from your desk except this quiz and a pen or pencil.** SHOW your work. Express your work in a neat and coherent manner. BOX your answer.

On planet Gzyx, a ball dropped from a height of 20 ft hits the ground in 2 seconds. If a ball is dropped from the top of a 200-ft-tall building on Gzyx, how long will it take to hit the ground? With what speed will it hit?

**ANSWER:** Let  $x(t)$  be the height of the ball above the ground at time  $t$ . Measure  $t$  in seconds and  $x$  in feet. We assume that  $x''(t) = -k$  for some positive constant  $k$ . For the first event, we have  $x(0) = 20$ ,  $x'(0) = 0$ , and  $x(2) = 0$ . For the second event, we have  $x(0) = 200$  and  $x'(0) = 0$ . We want to find  $t_1$  with  $x(t_1) = 0$ . We also want to find  $x'(t_1)$ .

We first think about the first event. Integrate to learn  $x'(t) = -kt + C_1$ . Plug in  $x'(0) = 0$  to learn that  $C_1 = 0$ . Integrate again to learn  $x(t) = -kt^2/2 + C_2$ . Plug in  $x(0) = 20$  to learn  $C_2 = 20$ . So,  $x(t) = -kt^2/2 + 20$ . Plug in  $x(2) = 0$  to learn  $k = 10$ .

Now turn to the second event. Integrate twice and evaluate the constants to learn that  $x'(t) = -kt$  and  $x(t) = -kt^2/2 + 200$ ; with  $k = 10$ ; so,  $x(t) = -5t^2 + 200$ . Solve  $0 = x(t_1) = -5t_1^2 + 200$  to learn that  $t_1 = \sqrt{40} = 2\sqrt{10}$  and  $x'(t_1) = -10t_1 = -20\sqrt{10}$ .

It takes the second ball  $2\sqrt{10}$  seconds to hit the ground. The ball is traveling downward at the speed  $20\sqrt{10}$  feet per second when it hits the ground.