PRINT Your Name:__________________________________

Quiz 9 on March 22, 2012

Each quiz is worth 5 points. **Remove EVERYTHING from your desk except this quiz and a pen or pencil.** SHOW your work. Express your work in a neat and coherent manner.

Solve the Initial Value Problem

$$x'' + 10x' + 125x = 0, \quad x(0) = 6,\ x'(0) = 50.$$

If possible, put your answer in the form $x(t) = C_1 e^{-pt}\cos(\omega_1 t - \alpha_1)$.

Answer. We try $x = e^{rt}$. We immediately are faced with solving the characteristic equation:

$$r^2 + 10r + 125 = 0.$$

So, $r = \frac{-10\pm\sqrt{100-500}}{2} = \frac{-10\pm 20i}{2} = -5\pm 10i$. So $x = c_1e^{-5t}\cos 10t + c_2e^{-5t}\sin 10t$ is the general solution of the differential equation. We now plug in the Initial Conditions to determine the values of c_1 and c_2. We calculate

$$x' = e^{-5t}(-10c_1\sin 10t - 5c_1\cos 10t + 10c_2\cos 10t - 5c_2\sin 10t).$$

We have $6 = x(0) = c_1$ and $50 = x'(0) = -5c_1 + 10c_2$; so, $c_1 = 6$ and $c_2 = 8$. The solution is

$$x(t) = e^{-5t}(6\cos 10t + 8\sin 10t) = 10e^{-5t}(\tfrac{6}{10}\cos 10t + \tfrac{8}{10}\sin 10t).$$

We have arranged the numbers so that $(\frac{6}{10}, \frac{8}{10})$ is a point on the unit circle. Let $\alpha = \arccos(\frac{6}{10})$. It follows that $\sin\alpha = \frac{8}{10}$. We have

$$x(t) = 10e^{-5t}(\cos\alpha\cos 10t + \sin\alpha\sin 10t) = 10e^{-5t}\cos(\alpha - 10t) = 10e^{-5t}\cos(10t - \alpha).$$

Our answer is

$$\boxed{x(t) = 10e^{-5t}\cos(10t-\alpha),\ \text{where } \alpha = \arccos(\tfrac{3}{5}).}$$