## Math 242, Final Exam, Fall, 2023

**You should KEEP this piece of paper.** Write everything on the **blank paper provided**. Return the problems **in order** (use as much paper as necessary), use **only one side** of each piece of paper. Number your pages and write your name on each page. Take a picture of your exam (for your records) just before you turn the exam in. I will e-mail your grade and my comments to you. I will keep your exam. **Fold your exam in half** before you turn it in.

The exam is worth 100 points. Each problem is worth 10 points. **Make your work coherent, complete, and correct.** Please $\boxed{CIRCLE}$ your answer. Please **CHECK** your answer whenever possible.

**No Calculators, Cell phones, computers, notes, etc.**

(1) Newton's Law of Cooling states that the rate of change with respect to time of the temperature $T(t)$ of an object is proportional to the difference between $T$ and the temperature $A$ of the surrounding medium. A four pound roast, initially at $50°$F, is placed in a $375°$F oven at 5:00 P.M. After 75 minutes it is found that the temperature $T(t)$ of the roast is $125°$F. When will the roast be $150°$F?

(2) A 120-gallon tank initially contains 90 lb of salt dissolved in 90 gal of water. Brine containing 2 lb/gal of salt flows into the tank at the rate of 4 gal/min, and the well stirred mixture flows out of the tank at the rate of 3 gal/min. Set up an initial Value problem for the the number of pounds $x(t)$ of salt in the tank at time $t$ for $0 \le t \le 30$, but **DO NOT SOLVE** the Initial Value Problem.

(3) Find the general solution of $(x^2 + 1)\frac{dy}{dx} + 3xy = 6x$.

(4) Find the general solution of $(x + y)\frac{dy}{dx} = x - y$.

(5) Find a particular solution of $y'' - y' - 6y = 2\sin 3x$. (In this problem $y$ is a function of $x$.)

(6) Find the general solution of $y'' - 4y' + 5y = 0$. (In this problem $y$ is a function of $x$.)

(7) Find the Laplace transform of $\sin^2 x$.

(8) Use Laplace transforms to solve the Initial Value Problem

$$x'' - x' - 6x = 0, \quad x(0) = 2, \quad x'(0) = 1.$$

In this problem $x$ is a function of $t$.

**PLEASE TURN OVER.**

(9) Find the Laplace transform of

$$f(t) = \begin{cases} t & \text{if } 0 \le t \le 1 \\ 0 & \text{if } 1 < t. \end{cases}$$

(10) Solve the Initial Value problem

$$\frac{dx}{dt} = x - 3, \quad x(0) = x_0.$$

Graph the solution of the Initial Value Problem for a few different choices of $x_0$.