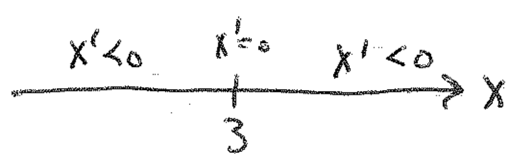
5. **Consider the Differential Equation** $\frac{dx}{dt} = -(3-x)^2$ **.**

(a) **Find all equilibrium solutions** $x(t) = x_e$ **for all** t **for some constant** x_e **.**

Solve $-(3-x)^2 = 0$ to see that $x = 3$ is the only equilibrium solution for the given DE.

(b) **For each equilibrium solution** $x(t) = x_e$ **of the DE, answer the following questions:**

Draw the phase diagram:



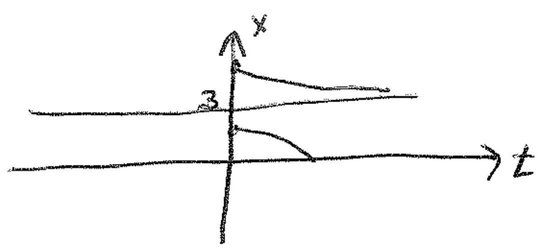
(i) **If** $x(0)$ **is a little less than** x_e **, does the corresponding solution** $x(t)$ **head toward or away from the equilibrium solution** $x = x_e$ **.**

If $x(0)$ is a little less than 3, then we look at the phase diagram to see that $x' < 0$, so $x(t)$ is a decreasing function. Thus, $x(t)$ heads away from 3.

(ii) **If** $x(0)$ **is a little more than** x_e **, does the corresponding solution** $x(t)$ **head toward or away from the equilibrium solution** $x = x_e$ **.**

If $x(0)$ is a little more than 3, then we look at the phase diagram to see that $x' < 0$, so $x(t)$ is a decreasing function. Thus, $x(t)$ heads toward 3.

(c) **Sketch a few solutions of the DE.**



(d) **Solve the DE.**

Separate the variables and integrate to see that $\int \frac{dx}{(3-x)^2} dx = -\int dt$. So, $\frac{1}{3-x} = -t + C$. We solve for x: $\frac{1}{-t+C} = 3 - x$; or $x = 3 - \frac{1}{-t+C}$; or $\boxed{x = 3 + \frac{1}{t-C}.}$ By the way, this answer checks. We compute $\frac{dx}{dt} = -\frac{1}{(t-C)^2}$. On the other hand, $-(3-x)^2 = -\left(-\frac{1}{t-C}\right)^2$ and these are the same. Of course the hyperbolas $x = 3 + \frac{1}{t-C}$ look like the pictures given below and this agrees with our expectations.