Math 242, Exam 1, Spring 2012

Write everything on the blank paper provided. **You should KEEP this piece of paper.** If possible: turn the problems in order (use as much paper as necessary), use only one side of each piece of paper, and leave 1 square inch in the upper left hand corner for the staple. If you forget some of these requests, don't worry about it – I will still grade your exam.

The exam is worth 50 points. There are **5** problems **ON TWO SIDES**. Each problem is worth 10 points. SHOW your work. $\boxed{CIRCLE}$ your answer. **CHECK** your answer whenever possible.

No Calculators or Cell phones.

1. Solve $x^3 + 3y - x\frac{dy}{dx} = 0$. Express your answer in the form $y(x)$. **Check your answer.**

2. Solve $2xy\frac{dy}{dx} = 4x^2 + 3y^2$. Express your answer in the form $y(x)$. **Check your answer.**

3. A 200-gallon tank is full of a solution containing 25 pounds of salt. Starting at time $t = 0$, pure water is added to the tank at a rate of 10 gallons per minute, and the well-stirred solution is withdrawn at the same rate. Find the number of pounds of salt in the tank at time t.

4.
   (a) State the Existence and Uniqueness Theorem for first order differential equations.
   (b) What does the Existence and Uniqueness Theorem tell you about the Initial Value Problem
   $$(1 + x^2)y' = (1 + y)^2 \quad y(0) = 0?$$
   (c) Solve the Initial Value Problem of part (b).
   (d) What does the Existence and Uniqueness Theorem tell you about the Initial Value Problem
   $$(1 + x^2)y' = (1 + y)^2 \quad y(0) = -1?$$
   (e) Solve the Initial Value Problem of part (d).

5. Newton's law of cooling states that the rate at which an object cools is proportional to the difference in temperature between the object and the surrounding medium. Thus, if an object is taken from an oven at 400° F and left to cool in a room at 70° F, then its temperature T after t hours will satisfy the differential equation

$$\frac{dT}{dt} = k(T - 70).$$

If the temperature fell to 200° F after one hour, what will it be after 4 hours? (You may leave "ln" in your answer.)