Math 242, Exam 1, Spring 2010

Write everything on the blank paper provided. **You should KEEP this piece of paper.** If possible: turn the problems in order (use as much paper as necessary), use only one side of each piece of paper, and leave 1 square inch in the upper left hand corner for the staple. If you forget some of these requests, don't worry about it – I will still grade your exam.

The exam is worth 50 points. There are **5** problems **ON TWO SIDES**. Each problem is worth 10 points. SHOW your work. $\boxed{CIRCLE}$ your answer. **CHECK** your answer whenever possible.

No Calculators or Cell phones.

1. Solve $yy' + x = \sqrt{x^2 + y^2}$. Express your answer in the form $y(x)$. **Check your answer.**

2. Solve $y' = y + y^3$. Express your answer in the form $y(x)$. **Check your answer.**

3. A tank contains 1000 liters (L) of a solution consisting of 100 kg of salt dissolved in water. Pure water is pumped into the tank at the rate of 5 L/s, and the mixture — kept uniform by stirring — is pumped out at the same rate. How long will it be until only 10 kg of salt remains in the tank?

4.
   (a) State the Existence and Uniqueness Theorem for first order differential equations.
   (b) What does the Existence and Uniqueness Theorem tell you about the Initial Value Problem

$$(1 + x^2)y' = (2 + y)^2 \quad y(0) = 0?$$

   (c) Solve the Initial Value Problem of part (b).
   (d) What does the Existence and Uniqueness Theorem tell you about the Initial Value Problem

$$(1 + x^2)y' = (2 + y)^2 \quad y(0) = -2?$$

   (e) Solve the Initial Value Problem of part (d).

5. When the brakes are applied to a certain car, the acceleration of the car is $-k$ m/s^2 for some positive constant k. Suppose that the car is traveling at the velocity v_0 m/s when the brakes are first applied and that the brakes continue to be applied until the car stops.
   (a) Find the distance that the car travels between the moment that the brakes are first applied and the moment when the car stops. (Your answer will be expressed in terms of k and v_0.)
   (b) How does the stopping distance change if the initial velocity is changed to $3v_0$?