Math 242, Exam 1, Fall, 2023

You should KEEP this piece of paper. Write everything on the **blank paper provided**. Return the problems **in order** (use as much paper as necessary), use **only one side** of each piece of paper. Number your pages and write your name on each page. Take a picture of your exam (for your records) just before you turn the exam in. I will e-mail your grade and my comments to you. I will keep your exam. **Fold your exam in half** before you turn it in.

The exam is worth 50 points. Each problem is worth 10 points. **Make your work coherent, complete, and correct.** Please $\boxed{CIRCLE}$ your answer. Please **CHECK** your answer whenever possible.

The solutions will be posted later today.

No Calculators, Cell phones, computers, notes, etc.

(1) A population of 80 cougars decreases at a rate of 5% per year. How many cougars will there be after 6 years?

(2) **Set this problem up completely. DO NOT SOLVE IT.** A 120-gallon tank initially contains 90 pounds of salt dissolved in 90 gallons of water. Brine containing 2 pounds/gallon of salt flows into the tank at the rate of 4 gallons per minute, and the well-stirred mixture flows out of the tank at the rate of 3 gallons per minute. How much salt does the tank contain when it is full?

(3) At time zero an obect has position x_0 and velocity v_0. Suppose that the object moves through a resisting medium with resistance proportional to its velocity v, so that $\frac{dv}{dt} = -kv$. Find the velocity and position of the object at time t.

(4) Solve the Differential Equation $xy' + 2y = 6x^2\sqrt{y}$.

PLEASE TURN OVER.

(5) The Logistic Equation is $\frac{dP}{dt} = kP(M - P)$, where k and M are positive constants. The solution of the Logistic Equation is

$$P(t) = \frac{MP(0)}{P(0) + (M - P(0))e^{-kMt}}.$$

Recall that if a population $P(t)$ satisfies the logistic equation

$$\frac{dP}{dt} = aP - bP^2,$$

where $B = aP$ is the time rate at which births occur and $D = bP^2$ is the rate at which deaths occur, then the limiting population is

$$M = \lim_{t \to \infty} P(t) = \frac{B(0)P(0)}{D(0)}.$$

Consider a rabbit population $P(t)$ which satisfies the logistic equation. If the initial population is 240 rabbits and there are 9 births per month and 12 deaths per month occurring at time $t = 0$, how many months does it take for $P(t)$ to reach 105% of the limiting population M?