

Math 242, Final Exam, Spring, 2024

You should KEEP this piece of paper. Write everything on the **blank paper provided**. Return the problems **in order** (use as much paper as necessary), use **only one side** of each piece of paper. Number your pages and write your name on each page. Take a picture of your exam (for your records) just before you turn the exam in. I will e-mail your grade and my comments to you. **Fold your exam in half** before you turn it in.

The exam is worth 100 points. Each problem is worth 10 points. **Make your work coherent, complete, and correct.** Please CIRCLE your answer. Please **CHECK** your answer whenever possible.

The solutions will be posted later today.

No Calculators, Cell phones, computers, notes, etc.

- (1) Solve the Initial Value problem

$$\frac{dx}{dt} = x - 5, \quad x(0) = x_0.$$

Graph the solution of the Initial Value Problem for a few different choices of x_0 .

- (2) Find $\mathcal{L}^{-1}\left(\frac{s+8}{s^2+4s+13}\right)$.
- (3) Use the method of Laplace transforms to solve the Initial Value Problem

$$x'' - x' - 6x = 0, \quad x(0) = 2, \quad \text{and} \quad x'(0) = -1.$$

- (4) Find the general solution of $xy' + 3y = 2x^5$. (In this problem $y = y(x)$.)
- (5) Find the general solution of $xyy' = x^2 + 3y^2$. (In this problem $y = y(x)$.)
- (6) Find the general solution of $y'' - 4y' + 13y = 0$. (In this problem $y = y(x)$.)
- (7) Newton's Law of Cooling states that the rate at which an object cools is proportional to difference between the temperature of the object and the temperature of the surrounding medium. At time zero a roast, with a temperature of 375 degrees F, is taken from the oven and placed in a room with temperature 70 degrees F. Twenty five minutes later, the temperature of the roast is 225 degrees F. When will the temperature of the roast reach 125 degrees?
- (8) Find a particular solution of $y'' - 3y' + 2y = e^x$. (In this problem $y = y(x)$.)
- (9) Find a particular solution of $y'' + 3y' + 2y = x^2$. (In this problem $y = y(x)$.)

Please turn over.

- (10) **Set up, but do not solve, an Initial Value Problem.** A 150-gallon tank initially contains 110 pounds of salt dissolved in 80 gallons of water. Brine containing 4 pounds per gallon of salt flows into the tank at the rate of 5 gallons per minute, and the well-stirred mixture flows out of the tank at the rate of 2 gallons per minute. How much salt is in the tank after t minutes?