

**Problem 21 in Section 7.2.** Find the inverse Laplace transform of

$$F(s) = \frac{1}{s^2(s^2 + 1)}$$

**Solution.** We use  $\mathcal{L}^{-1}\left(\frac{F(s)}{s}\right) = \int_0^t \mathcal{L}^{-1}(F(s))|_{\tau} d\tau$  twice. We compute

$$\begin{aligned} \mathcal{L}^{-1}\left(\frac{1}{s^2(s^2 + 1)}\right) &= \int_0^t \mathcal{L}^{-1}\left(\frac{1}{s(s^2 + 1)}\right)\Big|_{\tau} d\tau \\ &= \int_0^t \int_0^{\tau} \mathcal{L}^{-1}\left(\frac{1}{s^2 + 1}\right)\Big|_{\theta} d\theta d\tau \\ &= \int_0^t \int_0^{\tau} \sin(\theta) d\theta d\tau \\ &= \int_0^t -\cos(\theta)\Big|_0^{\tau} d\tau \\ &= \int_0^t (-\cos(\tau) + 1) d\tau \\ &= (-\sin \tau + \tau)\Big|_0^t \\ &= \boxed{-\sin t + t}. \end{aligned}$$