Problem 17 in Section 7.2. Find the inverse Laplace transform of

$$F(s) = \frac{1}{s(s-3)}$$

Solution. We use $\mathcal{L}^{-1}(\frac{F(s)}{s}) = \int_0^t \mathcal{L}^{-1}(F(s))|_\tau d\tau$. We compute

$$\begin{aligned}
\mathcal{L}^{-1}\left(\frac{1}{s(s-3)}\right) &= \mathcal{L}^{-1}\left(\frac{\frac{1}{(s-3)}}{s}\right) \\
&= \int_0^t \mathcal{L}^{-1}\frac{1}{(s-3)}|_\tau d\tau \\
&= \int_0^t e^{3\tau} d\tau \\
&= \frac{1}{3}e^{3\tau}\Big|_{\tau=0}^{\tau=t} \\
&= \boxed{\frac{1}{3}(e^{3t}-1)}.
\end{aligned}$$