Problem 11 in Section 7.1. Compute $\mathcal{L}(f(t))$ for $f(t) = \sqrt{t} + 3t$.

Solution. Use the second page of the fact sheet about Laplace Transforms.

$$\begin{aligned}\mathcal{L}(f(t)) &= \mathcal{L}(\sqrt{t} + 3t) \\ &= \mathcal{L}(\sqrt{t}) + 3\mathcal{L}(t)\end{aligned}$$

The fact sheet says that $\mathcal{L}(t^a) = \frac{\Gamma(a+1)}{s^{a+1}}$. It follows that

$$\Gamma(t^{1/2})\frac{\Gamma(\frac{1}{2}+1)}{s^{\frac{1}{2}+1}} = \frac{\frac{1}{2}\Gamma(\frac{1}{2})}{s^{3/2}} = \frac{\sqrt{\pi}}{2s^{3/2}}$$

The fact sheet also says that $\Gamma(t) = \frac{1}{s^2}$. We conclude that

$$\boxed{\mathcal{L}(f(t)) = \frac{\sqrt{\pi}}{2s^{3/2}} + \frac{3}{s^2}.}$$