Problem 2 in Section 3.5. Find a particular solution of

$$y'' - y' - 2y = 3x + 4.$$

Solution. We try $y = Ax + B$. Plug

$$y = Ax + B$$
$$y' = A$$
$$y'' = 0$$

into $y'' - y' - 2y = 3x + 4$ and obtain

$$0 - A - 2(Ax + B) = 3x + 4$$

In other words,

$$-2Ax + (-2B - A) = 3x + 4.$$

Equate the corresponding coefficients. We solve

$$\begin{cases} -2A = 3 \\ (-2B - A) = 4. \end{cases}$$

We see that $A = \frac{3}{-2}$ and $-2B = 4 - \frac{3}{2}$, Thus, $B = \frac{-1}{2}\frac{5}{2} = -\frac{5}{4}$.

We conclude that $\boxed{y = \frac{3}{-2}x - \frac{5}{4}}$ is a particular solution of $y'' - y' - 2y = 3x + 4$.

Check. We plug

$$y = \frac{3}{-2}x - \frac{5}{4}$$
$$y' = \frac{3}{-2}$$
$$y'' = 0$$

into $y'' - y' - 2y$ and obtain

$$0 - \frac{3}{-2} - 2(\frac{3}{-2}x - \frac{5}{4}) = 3x + \frac{3}{2} + \frac{5}{2} = 3x + 4.\checkmark$$