Problem 1 in Section 3.3. Find the general solution of $y'' - 4y = 0$.

Solution. We try $y = e^{rx}$. We plug y, $y' = re^{rx}$ and $y'' = r^2e^{rx}$ into the Differential Equation. We want

$$r^2e^{rx} - 4e^{rx} = 0.$$

We want $e^{rx}(r^2 - 4) = 0$. If a product is zero, one of the factors must be zero. The function e^{rx} is never zero; so we want $r^2 - 4 = 0$. In other words, $r = 2$ or $r = -2$. The general solution of $y'' - 4y = 0$ is $\boxed{y = c_1e^{2x} + c_2e^{-2x}}$.

Check. We plug

$$\begin{aligned} y &= c_1e^{2x} + c_2e^{-2x} \\ y' &= 2c_1e^{2x} - 2c_2e^{-2x} \\ y'' &= 4c_1e^{2x} + 4c_2e^{-2x} \end{aligned}$$

into $y'' - 4y$ and obtain

$$\begin{aligned} &\left(4c_1e^{2x} + 4c_2e^{-2x}\right) - 4\left(c_1e^{2x} + c_2e^{-2x}\right) \\ &= (4c_1 - 4c_1)e^{2x} + (4c_2 - 4c_2)e^{-2x} = 0.\checkmark \end{aligned}$$