

Problem 4 in Section 3.2. Find a nontrivial linear combination of $f(x) = 17$, $g(x) = 2 \sin^2 x$, and $h(x) = 3 \cos^2 x$ which is the constant function zero.

Solution. We want numbers (at least one not zero) a_1, a_2, a_3 , with $a_1 f(x) + a_2 g(x) + a_3 h(x)$ equal to the constant function 0. We take advantage of the Trig identity $\sin^2 x + \cos^2 x = 1$. So $\frac{1}{2}g(x) + \frac{1}{3}h(x)$ is the constant function 1. Thus

$$-\frac{1}{17}f(x) + \frac{1}{2}g(x) + \frac{1}{3}h(x)$$

is the constant function 0.