Problem 21 in Section 3.2. The problem tells us that $y_{\text{homog}} = c_1 \cos x + c_2 \sin x$ is the general solution of the homogeneous problem $y'' + y = 0$ and $y_{\text{partic}} = 3x$ is a particular solution of the Differential Equation $y'' + y = 3x$. The problem tells us to find the solution of the Initial Value Problem

$$y'' + y = 3x, \quad y(0) = 2, \quad \text{and} \quad y'(0) = -2.$$

Solution. The general solution of $y'' + y = 3x$ is

$$y = c_1 \cos x + c_2 \sin x + 3x.$$

We must evaluate the constants. We calculate

$$y' = -c_1 \sin x + c_2 \cos x + 3$$

Plug $x = 0$ into y and y'. We must solve

$$\begin{aligned} 2 &= c_1 \cos(0) + c_2 \sin(0) + 3(0) \\ -2 &= -c_1 \sin(0) + c_2 \cos(0) + 3 \end{aligned}$$

We must solve

$$\begin{aligned} 2 &= c_1 \\ -2 &= c_2 + 3 \end{aligned}$$

We conclude that $c_1 = 2$ and $c_2 = -5$. The solution of the Initial Value Problem is

$$\boxed{y = 2\cos x - 5\sin x + 3x.}$$

Check. We compute

$$\begin{aligned} y &= 2\cos x - 5\sin x + 3x \\ y' &= -2\sin x - 5\cos x + 3 \\ y'' &= -2\cos x + 5\sin x \end{aligned}$$

Plug y, y', and y'' into the left side of the Differential Equation. We see that

$$y'' + y = (-2\cos x + 5\sin x) + (2\cos x - 5\sin x + 3x) = 3x.\checkmark$$

We compute $y(0) = 2(1) - 5(0) + 3(0) = 2\checkmark$ and $y'(0) = -2\sin(0) - 5\cos(0) + 3 = -5 + 3 = -2\checkmark$. Our proposed answer does everything it is supposed to do. Our answer is correct.