Math 241 Exam 3 Summer 2002

PRINT Your Name:________________________________

There are 10 problems on 6 pages. Each problem is worth 5 points. SHOW your work. $\boxed{CIRCLE}$ your answer. **NO CALCULATORS!**

I will put your exam outside my office door after I have graded it. You may pick it up any time before class on Monday. If I know your e-mail address, I will e-mail your score on Exam 3 to you.

1. Let $f(x,y)=2x^2y^3-x^3y^5$. Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$.

2. Sketch and label the level sets for levels 1, 0, -1, for the function $f(x,y)=x^2-y^2$.

3. Graph and name $z=x^2+y^2$ in 3-space.

4. Graph and name $z^2=x^2+y^2$ in 3-space.

5. Find the equation of the plane tangent to $z^2=x^2+y^2$ at the point $(3,4,5)$.

6. Consider the curve whose position vector is
$$\overrightarrow{\boldsymbol{r}}(t)=2t^2\,\overrightarrow{\boldsymbol{i}}-t^3\,\overrightarrow{\boldsymbol{j}}+\tfrac{4}{t}\,\overrightarrow{\boldsymbol{k}}.$$
Find the equations of the line tangent to this curve at $t=1$.

7. Find the directional derivative $D_{\overrightarrow{\boldsymbol{u}}}f$ at $(1,2)$ for the function $f(x,y)=3x^2y$ in the direction of the unit vector $\frac{3}{5}\,\overrightarrow{\boldsymbol{i}}+\frac{4}{5}\,\overrightarrow{\boldsymbol{j}}$.

8. Find the length of the curve whose position vector is
$$\overrightarrow{\boldsymbol{r}}(t)=t^2\,\overrightarrow{\boldsymbol{i}}-2t^3\,\overrightarrow{\boldsymbol{j}}+6t^3\,\overrightarrow{\boldsymbol{k}},$$
for $0\le t\le 1$.

9. The temperature of a plate at the point (x,y) is $T(x,y)=xy$.
   (a) Draw and label the level sets $T=0$, $T=1$, $T=-1$, $T=2$, and $T=-2$.
   (b) A heat seeking particle always moves in the direction of the greatest increase in temperature. Place such a particle on your answer to (a) at the point $(1,-2)$. Draw the path of the particle.
   (c) Find the equation which gives the path of the particle of part (b).

10. The position of a moving particle at time t is given by the position vector
$$\overrightarrow{\boldsymbol{r}}(t)=2\cos t\,\overrightarrow{\boldsymbol{i}}-3\sin^2 t\,\overrightarrow{\boldsymbol{j}}.$$
   (a) Graph the path of the object.
   (b) Eliminate the parameter and express the path of the object in cartesian coordinates.
   (c) Which point on the curve corresponds to $t=\frac{\pi}{3}$?
   (d) Draw $\overrightarrow{\boldsymbol{v}}(\frac{\pi}{3})$. Point the tail on your answer to (c).
   (e) Draw $\overrightarrow{\boldsymbol{a}}(\frac{\pi}{3})$. Point the tail on your answer to (c).