Math 241 Final Exam Spring 2008

Please leave room in the upper left corner for the staple.

TAKE THESE QUESTIONS HOME WITH YOU WHEN YOU LEAVE.

Write your answers as legibly as you can on the blank sheets of paper provided. Use only **one side** of each sheet. Be sure to number your pages. Put your solution to problem 1 first, and then your solution to number 2, etc.; although, by using enough paper, you can do the problems in any order that suits you.

There are 10 problems. Each problem is worth 10 points. The exam is worth 100 points. SHOW your work. Make your work be coherent and clear. Write in complete sentences whenever this is possible. $\boxed{CIRCLE}$ your answer. **CHECK** your answer whenever possible. **No Calculators.**

1. Let P be the point $P = (1, 2, 3)$ and let $\mathfrak{P}$ be the plane $2x - 2y + z = 4$.
   a. What is the distance from P to $\mathfrak{P}$?
   b. What is the point on $\mathfrak{P}$ which is nearest to P?
   c. What is the equation of the line which is perpendicular to $\mathfrak{P}$ and passes through P?

2. Consider the curve C which starts and stops at the point $(1, 0)$. The first leg of the curve goes along the x-axis to $(2, 0)$. The second leg of the curve goes counter-clock-wise along the circle of radius 2 and center $(0, 0)$ to $(-2, 0)$. The third leg of the curve goes along the x-axis to $(-1, 0)$. The fourth and final leg of the curve goes clock-wise along the circle of radius 1 and center $(0, 0)$ to $(1, 0)$. Find
$$\int_C (3y + x)dx + (8x - 15y)dy.$$

3. Consider the curve C which starts at the point $(1, 0)$. The first leg of C is the line segment to $(3, 5)$. The second leg of C is the line segment to $(5, -6)$. The final leg of C is the line segment to $(7, 4)$. Find
$$\int_C 3ydx + (3x + 4y)dy.$$

4. Find the maximum and minimum values of $f(x, y) = 4x^3 + y^2$ subject to the constraint $2x^2 + y^2 = 1$.

5. Find the equation of the line tangent to the curve $\vec{\boldsymbol{r}}(t) = t\,\vec{\boldsymbol{i}} + t^2\,\vec{\boldsymbol{j}}$ at the point $P = (2, 4)$.

6. Compute
$$\int_0^4 \int_{\sqrt{y}}^2 e^{x^3}\, dx dy.$$

7. Find the volume of the wedge cut from the cylinder $4x^2 + y^2 = 9$ by the planes $z = 0$ and $z = y + 3$.

8. **Make sure your answer is correct.** Find the equation of the plane that contains the points $(1, 7, 1)$, $(4, 1, 1)$, and $(1, 1, 3)$.

9. **Make sure your answer is correct.** Let $\vec{a} = 1\vec{i} + 2\vec{j} - 1\vec{k}$ and $\vec{b} = 2\vec{i} + 7\vec{j} - 2\vec{k}$. Find vectors $\vec{u}$ and $\vec{v}$ with $\vec{b} = \vec{u} + \vec{v}$, $\vec{u}$ parallel to $\vec{a}$, and $\vec{v}$ perpendicular to $\vec{a}$. (Every number in the answer is an integer. If you have fractions, either you can rid of them or you have made a mistake.)

10. Find the mass of a spherical solid of radius a if the density is proportional to the distance from the center. (Let k be the constant of proportionality.)