No calculators, cell phones, computers, notes, etc.

Circle your answer. Make your work correct, complete, and coherent.

The quiz is worth 5 points. The solutions will be posted on my website later today.

Quiz 6, April 11, 2019

Find the volume of the region between the planes $x + y + 2z = 2$ and $2x + 2y + z = 4$ in the first octant.

$z = 4 - 2x - 2y$

$z = \dfrac{2 - x - y}{2}$

$Vol = \displaystyle\iint\int_{bottom}^{top} dz\, dA$

fattest Cross Section

$= \displaystyle\iint \int_{\frac{2-x-y}{2}}^{4-2x-2y} dz\, dA = \int_0^2 \int_0^{2-x} \int_{\frac{2-x-y}{2}}^{4-2x-2y} dz\, dy\, dx$

$= \displaystyle\int_0^2 \int_0^{2-x} z\Big]_{\frac{2-x-y}{2}}^{4-2x-2y} dy\, dx = \int_0^2 \int_0^{2-x} (4-2x-2y) - \left(1 - \frac{x}{2} - \frac{y}{2}\right) dy\, dx$

$= \displaystyle\int_0^2 \int_0^{2-x} 3 - \frac{3}{2}x - \frac{3}{2}y \; dy\, dx = \int_0^2 3y - \frac{3}{2}xy - \frac{3}{4}y^2 \Big]_0^{2-x} dx$

$= \displaystyle\int_0^2 3(2-x) - \frac{3}{2}x(2-x) - \frac{3}{4}(2-x)^2 \, dx$

$= \displaystyle\int_0^2 3(2-x) - 3x + \frac{3}{2}x^2 - \frac{3}{4}(2-x)^2 \, dx$

$= -\dfrac{3(2-x)^2}{2} - \dfrac{3x^2}{2} + \dfrac{x^3}{2} + \dfrac{1}{4}(2-x)^3 \Big]_0^2$

$= -6 + 4 - (-6 + 2) = \boxed{2}$