

Math 241, Spring 1998, exam 3

PRINT Your Name: _____

There are 10 problems on 5 pages. Each problem is worth 10 points. SHOW your work. CIRCLE your answer. **NO CALCULATORS!** CHECK your answer, whenever possible.

1. Let $f(x, y, z) = xz \ln(x + y + z)$. Find $\vec{\nabla} f$.
2. Find the equation of the plane tangent to $z = x^2 + y^2$ at $(1, 1, 2)$.
3. Suppose that $z = f(x, y)$, and x and y are written polar coordinates (that is, $x = r \cos \theta$ and $y = r \sin \theta$). Express $\frac{\partial z}{\partial \theta}$ in terms of $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.
4. Find the directional derivative of $f(x, y, z) = xy + z^2$ at $(1, 1, 1)$ in the direction toward $(5, -3, 3)$.
5. Let $f(x, y) = \frac{xy^2}{3x^2 + 2y^4}$.
 - (a) Calculate the limit of $f(x, y)$ as $(x, y) \rightarrow (0, 0)$ along every straight line of the form $y = mx$.
 - (b) Calculate the limit of $f(x, y)$ as $(x, y) \rightarrow (0, 0)$ along the parabola $x = y^2$.
 - (c) What is $\lim_{(x,y) \rightarrow (0,0)} f(x, y)$?
6. Identify all local extreme points and all saddle points of $f(x, y) = x^3 + y^3 - 6xy$.
7. Graph $z = 9 - y^2 - x^2$ in 3-space.
8. Graph and label the level sets $f = 0$, $f = 10$, and $f = 20$ for $f(x, y) = 2x^2 - y^2 + 10$.
9. Graph $y^2 - \frac{x^2}{4} - \frac{z^2}{9} = 1$ in 3-space.
10. Find the absolute extreme points of $f(x, y) = x^2 - 6x + y^2 - 8y + 7$ on $\{(x, y) \mid x^2 + y^2 \leq 1\}$.