

Please PRINT your name _____

No calculators, cell phones, computers, notes, etc.

Circle your answer. Make your work correct, complete and coherent.

The quiz is worth 5 points. The solutions will be posted on my website later today.

Quiz 6, November 20, 2018

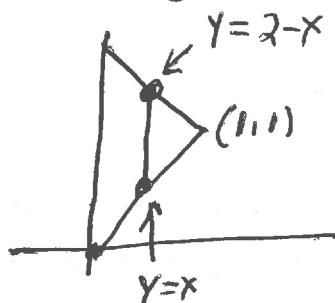
Find the volume of the region bounded above by the paraboloid $z = x^2 + y^2$ and below by the triangle enclosed by the lines $y = x$, $x = 0$, and $x + y = 2$ in the xy -plane.

ANSWER:

$$\text{Vol} = \iint_{\text{base}} \text{top} \, dA = \iint x^2 + y^2 \, dA$$

The intersection:

$$\begin{aligned} x &= 2-x \\ 2x &= 2 \\ x &= 1 \end{aligned}$$



$$= \int_0^1 \int_x^{2-x} (x^2 + y^2) \, dy \, dx = \int_0^1 \left[x^2 y + \frac{y^3}{3} \right]_x^{2-x} dx$$

$$= \int_0^1 x^2(2-x) + \frac{(2-x)^3}{3} - \left(x^3 + \frac{x^3}{3} \right) dx$$

$$= \int_0^1 2x^2 - \frac{7}{3}x^3 + \frac{(2-x)^3}{3} dx = \left[\frac{2x^3}{3} - \frac{7}{12}x^4 - \frac{(2-x)^4}{4 \cdot 3} \right]_0^1$$

$$= \frac{2}{3} - \frac{7}{12} - \frac{1}{12} + \frac{16}{12} = \frac{8-7-1+16}{12} = \frac{16}{12} = \boxed{\frac{4}{3}}$$