No calculators, cell phones, computers, notes, etc.

Circle your answer. Make your work correct, complete and coherent.

The quiz is worth 5 points. The solutions will be posted on my website later today.

Quiz 5, October 24, 2017, 1:15 class

An object is fired from the origin in the xy-plane at an angle α from the positive x-axis with an initial speed of v_0. The acceleration of the object is $-g\overrightarrow{\boldsymbol{j}}$. How high is the object when its x-coordinate is R?

Let $\overrightarrow{\boldsymbol{r}}(t) = x(t)\overrightarrow{\boldsymbol{i}} + y(t)\overrightarrow{\boldsymbol{j}}$ be the position vector of the object at time t. We are told that $\overrightarrow{\boldsymbol{r}}''(t) = -g\overrightarrow{\boldsymbol{j}}$, $\overrightarrow{\boldsymbol{r}}'(0) = v_0\cos\alpha\overrightarrow{\boldsymbol{i}} + v_0\sin\alpha\overrightarrow{\boldsymbol{j}}$, and $\overrightarrow{\boldsymbol{r}}(0) = 0\overrightarrow{\boldsymbol{i}} + 0\overrightarrow{\boldsymbol{j}}$. We integrate to learn $\overrightarrow{\boldsymbol{r}}'(t) = -gt\overrightarrow{\boldsymbol{j}} + \overrightarrow{c_1}$. Plug in $t = 0$ to learn

$$v_0\cos\alpha\overrightarrow{\boldsymbol{i}} + v_0\cos\alpha\overrightarrow{\boldsymbol{j}} = \overrightarrow{\boldsymbol{r}}'(0) = \overrightarrow{c_1}.$$

So,

$$\overrightarrow{\boldsymbol{r}}'(t) = v_0\cos\alpha\overrightarrow{\boldsymbol{i}} + (v_0\sin\alpha - gt)\overrightarrow{\boldsymbol{j}}.$$

Integrate again to learn

$$\overrightarrow{\boldsymbol{r}}(t) = (v_0\cos\alpha)t\overrightarrow{\boldsymbol{i}} + \big((v_0\sin\alpha)t - gt^2/2\big)\overrightarrow{\boldsymbol{j}} + \overrightarrow{c_2}.$$

Plug in $t = 0$ to learn

$$0 = \overrightarrow{\boldsymbol{r}}(0) = \overrightarrow{c_2}.$$

Thus,

$$\overrightarrow{\boldsymbol{r}}(t) = (v_0\cos\alpha)t\overrightarrow{\boldsymbol{i}} + \big((v_0\sin\alpha)t - gt^2/2\big)\overrightarrow{\boldsymbol{j}}.$$

The x-coordinate of the object is R when

$$(v_0\cos\alpha)t = R,$$

so $t = R/(v_0\cos\alpha)$. When the x-coordinate is R, the y coordinate is

$$y(R/(v_0\cos\alpha)) = (v_0\sin\alpha)(R/(v_0\cos\alpha)) - g\left(\frac{(R/(v_0\cos\alpha))^2}{2}\right)$$

$$= \boxed{R\tan\alpha - \frac{gR^2}{2v_0^2\cos^2\alpha}}.$$