Please PRINT your name ____

No calculators, cell phones, computers, notes, etc.

Circle your answer. Make your work correct, complete and coherent.

Please take a picture of your quiz (for your records) just before you turn the quiz in. I will e-mail your grade and my comments to you. I will keep your quiz.

The quiz is worth 5 points. The solutions will be posted on my website later today.

Quiz 4, February 22, 2023

A marble is fired from the origin with initial speed v_0 and launch angle α. The position vector of the marble at time t is $\vec{\boldsymbol{r}}(t)$. Assume that $\vec{\boldsymbol{r}}''(t) = -g\vec{\boldsymbol{j}}$ for some constant g.

(a) What is $\vec{\boldsymbol{r}}(t)$?
(b) When is the x-coordinate of the marble equal to R?
(c) What is the height of the marble at your answer to (b)?

Answer:

(a) Integrate to learn $\vec{\boldsymbol{r}}'(t) = -gt\vec{\boldsymbol{j}} + \vec{\boldsymbol{c}}_1$ for some constant vector $\vec{\boldsymbol{c}}_1$. Plug in $t = 0$ to obtain

$$v_0\cos\alpha\,\vec{\boldsymbol{i}} + v_0\sin\alpha\,\vec{\boldsymbol{j}} = \vec{\boldsymbol{r}}'(0) = -g(0)\vec{\boldsymbol{j}} + \vec{\boldsymbol{c}}_1 = \vec{\boldsymbol{c}}_1.$$

Thus,

$$\vec{\boldsymbol{r}}'(t) = v_0\cos\alpha\,\vec{\boldsymbol{i}} + (v_0\sin\alpha - gt\,\vec{\boldsymbol{j}}.$$

Integrate again to obtain

$$\vec{\boldsymbol{r}}(t) = (v_0\cos\alpha)t\,\vec{\boldsymbol{i}} + (v_0\sin\alpha)t - \frac{gt^2}{2}\vec{\boldsymbol{j}} + \vec{\boldsymbol{c}}_2,$$

for some constant vector $\vec{\boldsymbol{c}}_2$. Plug in $t = 0$ to obtain

$$-\vec{\boldsymbol{i}} + 0\vec{\boldsymbol{j}} = \vec{\boldsymbol{r}}(0) = (v_0\cos\alpha)0\,\vec{\boldsymbol{i}} + (v_0\sin\alpha)0 - \frac{g0}{2}\vec{\boldsymbol{j}} + \vec{\boldsymbol{c}}_2.$$

Thus $\vec{\boldsymbol{c}}_2 = 0$ and

$$\boxed{\vec{\boldsymbol{r}}(t) = (v_0\cos\alpha)t\,\vec{\boldsymbol{i}} + (v_0\sin\alpha t - \frac{gt^2}{2}\vec{\boldsymbol{j}}).}$$

(b) Observe that the x-coordinate of the position vector is equal to R when $R = (v_0\cos\alpha)t$ or

$$\boxed{\frac{R}{v_0\cos\alpha} = t.}$$

(c) When $t = \frac{R}{v_0 \cos\alpha}$, the height of the marble is

$$\begin{aligned} & v_0 \sin\alpha \frac{R}{v_0 \cos\alpha} - \frac{g(\frac{R}{v_0\cos\alpha})^2}{2} \\ & = \boxed{R\tan\alpha - \frac{gR^2}{2v_0^2\cos^2\alpha}.} \end{aligned}$$